NAME:

114 Spring 2011: Calculus

MIDTERM II : PRACTICE!

This practice exam is *not* multiple choice. Just work out these problems. Several of these problems come from old 114 exams. The length of this exam will be a full 80 minutes: be prepared to do lots of problems. Most will be on "the basics" but a few will dig deeper. Note that you will likely see problems on the real midterm which look nothing like what you see here!

PROBLEM 1: Compute the equation for the tangent plane to the surface

$$x^3 - xy^2 + 4xz = 8$$

at the point $(1, 2, 3)$.

PROBLEM 2: Use the chain rule to compute the derivative of $f(g(0, 0, 0))$, where

$$f\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \cos(uvw) \\ \sin(uvw) \\ \tan(u+v+w) \end{pmatrix} \quad ; \quad g\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+z \\ xyz \\ \pi e^{xyz} \end{pmatrix}$$

Hint: use matrices and matrix multiplication... Also: be careful, since $g(0, 0, 0) = (0, 0, \pi)$ — you must evaluate $[Df]$ at $g(0, 0, 0)$.

PROBLEM 3: Find the equation of the line tangent to the ellipse $2x^2 + y^2 = 3$ at $(1, 1)$.

PROBLEM 4: Compute the Taylor expansion of

$$f(x, y, z) = x^3 - 2xy + x^2 z + 7z$$

about the point $(1, -2, 1)$ up to and including terms of order two. (You are going to have to compute a lot of partial derivatives; evaluate at the point; then write a polynomial in the variable $(x - 1)$ and $(y + 2)$ and $(z - 1)$.) Now, tell me, at this point, which variable produces the largest change in f when varied?

PROBLEM 5: In which direction does the function $x^2 - y^2 - z^2$ **decrease** most rapidly at the point $(1, 1, 0)$?

PROBLEM 6: Find and classify all critical points of $x^3 - 12xy + 8y^3$.

PROBLEM 7: A silo must be built with a flat circular bottom, a cylindrical side, and a hemispherical top. The silo must hold 900π cubic feet of grain when totally filled. The cost of materials in dollars per square foot is 1 for the floor; 2 for the sides; and 5 for the roof. What are the most cost-effective silo dimensions?

PROBLEM 8: Taylor expand $\sin(y+z+e^{xy}-\cos(xz))$ about $(0,0,0)$ up to and including terms of order four. Do not compute partials – use standard Taylor expansions.

PROBLEM 9: Classify all critical points of the function

$$f(x,y)=\frac{8}{3}x^3+4y^3-x^4-y^4$$

Also, find the global maximum.

PROBLEM 10: For a triangle of side lengths a, b, and c; and perimeter $p=2s=a+b+c$, the area is equal to

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

(This is called *Heron's formula*, by the way...) Use a Lagrange multiplier to show that for fixed perimeter, the triangle with largest area is equilateral. (Hint: optimize A^2 to eliminate the radical.)

PROBLEM 11: The surface area of a cone of height h and radius r is $A=\pi r\sqrt{r^2+h^2}$. Use a linear approximation (i.e., Taylor expand to first order) to estimate the increase in A as (r,h) change from $(2,1)$ to $(1.98,1.03)$. You should not need a calculator for this.