NAME:

114 Spring 2011: Calculus

MIDTERM III : PRACTICE!

This practice exam is *not* multiple choice. Just work out these problems. The length of this exam will be a full 80 minutes: be prepared to do lots of problems. Most will be on "the basics" but a few will dig deeper. This practice exam contains a few more harder problems, to help you better prepare. Note that you will likely see problems on the real midterm which look nothing like what you see here!

PROBLEM 1: Find the general solution to $xy' - 2y = x^2$; then, if $y(1) = 3$, compute $y(e)$.

PROBLEM 2: What is the integrating factor for the differential equation

$$xy' - x^2 y = \cos x$$

Compute the integrating factor and show that multiplying by it allows one to integrate the left hand side. (Don't integrate the right hand side; or, rather, don't try!)

PROBLEM 3: Draw a picture of the domain over which the following integral computes area, then write a double integral that reverses the order of integration, and evaluate it.

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3}\, dy\, dx$$

PROBLEM 4: Find the volume of the region between the sphere $\rho = \cos\phi$ and the hemisphere $\rho = 2$, $z \geq 0$.

PROBLEM 5: (a) What proportion of the earth's surface lies above the 45° north latitude line? Assume the earth is a perfect sphere of radius R. (b) The earth is not really spherical but rather the spheroidal image of a perfect sphere under the matrix $\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where this is in (x, y, z) coords, z points toward the north pole, and λ is a constant (close to, but a little more than 1). Does your answer to part (a) change in this case? That is, does the proportion depend upon λ?

PROBLEM 6: Evaluate the integral

$$\int_0^\infty \int_0^\infty \frac{dx\, dy}{(1 + x^2 + y^2)^2}$$

PROBLEM 7: Compute the area inside the circle $r = 4\cos\theta$ but to the right of the line $r = \sec\theta$.

PROBLEM 8: Consider the unit cube ($-\frac{1}{2} \leq x_i \leq \frac{1}{2}$ for $i = 1 \ldots n$). centered at the origin. What is the average of the square of the distance-to-the-origin? Is there some value of n for which it is greater than one? Are you surprised at this?

PROBLEM 9: Find the volume of the region between the paraboloids $z = 5 - x^2 - y^2$ and $z = 4x^2 + 4y^2$.

PROBLEM 10: Use an appropriate change of coordinates to compute the integral

$$\int_0^{\frac{2}{3}} \int_y^{2-2y} (x + 2y)e^{y-x} dx\, dy$$

PROBLEM 11: A solid in the first octant is bounded by surfaces $z = 4 - x^2$ and $x = y^2$. Its density function is $\rho = xy$. Find the mass and the center of mass of the solid.

PROBLEM 12: Rewrite the double integral

$$\int_0^{\infty} \int_0^{x} e^{-sx} f(x - y, y) dy\, dx$$

under the change of coordinates $u = x - y$ and $v = y$. Does the new double integral look simpler? [This is an important computation in Laplace transforms...]