MATH 114 Quiz 1 Solution

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Notice: You should show your work to justify your answer. Each correct step will be rewarded. Problems are on **BOTH** sides of this paper.

Problem 1

Let P be the plane that contains the points (1,0,0), (0,2,0), and (0,0,3). What is the distance from P to the origin?

Solution:
First of all, we need to find the equation of the plane P. Denote $A(1,0,0), B(0,2,0), C(0,0,3)$. Then two vectors in the plane:

$$\begin{aligned} \vec{AB} &= -\mathbf{i} + 2\mathbf{j} \\ \vec{AC} &= -\mathbf{i} + 3\mathbf{k} \end{aligned}$$

The normal vector of plane P:

$$\begin{aligned} \mathbf{n} = \vec{AB} \times \vec{AC} &= \mathbf{i} \times \mathbf{i} - 3\mathbf{i} \times \mathbf{k} - 2\mathbf{j} \times \mathbf{i} + 6\mathbf{j} \times \mathbf{k} \\ &= 6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} \end{aligned}$$

So the equation of the plane is

$$6x + 3y + 2z - 6 = 0$$

The distance D from $(0,0,0)$ to this plane is (by formula 9 on page 837)

$$D = \frac{|6 \cdot 0 + 3 \cdot 0 + 2 \cdot 0 - 6|}{\sqrt{6^2 + 3^2 + 2^2}} = \frac{|-6|}{\sqrt{36 + 9 + 4}} = \frac{6}{\sqrt{49}} = \frac{6}{7}$$

Problem 2

A helix has parametric equation $\mathbf{r}(t) = (\cos t, \sin t, t)$. Find the arc length from $\mathbf{r}(0)$ to $\mathbf{r}(2\pi)$.

Solution:
The tangent vector to the curve $\mathbf{f}(t) = (\cos t, \sin t, t)$ is given by

$$\mathbf{r}'(t) = (-\sin t, \cos t, 1)$$

The time range is $t \in [0, 2\pi]$, along with formula 2 on page 866, the arc length is

$$L = \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2 + (1)^2} dt = \int_0^{2\pi} \sqrt{2} dt = 2\sqrt{2}\pi$$