

Science is what we understand well enough to explain to a computer. Art is everything else we do. During the past several years an important part of mathematics has been transformed from an Art to a Science: No longer do we need to get a brilliant insight in order to evaluate sums of binomial coefficients, and many similar formulas that arise frequently in practice; we can now follow a mechanical procedure and discover the answers quite systematically.

I fell in love with these procedures as soon as I learned them, because they worked for me immediately. Not only did they dispose of sums that I had wrestled with long and hard in the past, they also knocked off two new problems that I was working on at the time I first tried them. The success rate was astonishing.

In fact, like a child with a new toy, I can't resist mentioning how I used the new methods just yesterday. Long ago I had run into the sum $\sum_k \binom{2n-2k}{n-k} \binom{2k}{k}$, which takes the values 1, 4, 16, 64 for $n = 0, 1, 2, 3$ so it must be 4^n . Eventually I learned a tricky way to prove that it is, indeed, 4^n ; but if I had known the methods in this book I could have proved the identity immediately. Yesterday I was working on a harder problem whose answer was $S_n = \sum_k \binom{2n-2k}{n-k}^2 \binom{2k}{k}^2$. I didn't recognize any pattern in the first values 1, 8, 88, 1088, so I computed away with the Gosper-Zeilberger algorithm. In a few minutes I learned that $n^3 S_n = 16(n - \frac{1}{2})(2n^2 - 2n + 1)S_{n-1} - 256(n-1)^3 S_{n-2}$.

Notice that the algorithm doesn't just verify a conjectured identity " $A = B$ ". It also answers the question "What is A ?", when we haven't been able to formulate a decent conjecture. The answer in the example just considered is a nonobvious recurrence from which it is possible to rule out any simple form for S_n .

I'm especially pleased to see the appearance of this book, because its authors have not only played key roles in the new developments, they are also master expositors of mathematics. It is always a treat to read their publications, especially when they are discussing really important stuff.

Science advances whenever an Art becomes a Science. And the state of the Art advances too, because people always leap into new territory once they have understood more about the old. This book will help you reach new frontiers.

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