Preprints:

1. C. He, P. Petersen, & W. Wylie, *On the classification of warped product Einstein metrics.* arXiv:1010.5488v1

Abstract: In this paper we take the perspective introduced by Case-Shu-Wei of studying warped product Einstein metrics through the equation for the Ricci curvature of the base space. They call this equation on the base the m-Quasi Einstein equation, but we will also call it the $(\lambda, n+m)$-Einstein equation. In this paper we extend the work of Case-Shu-Wei and some earlier work of Kim-Kim to allow the base to have non-empty boundary. This is a natural case to consider since a manifold without boundary often occurs as a warped product over a manifold with boundary, and in this case we get some interesting new canonical examples. We also derive some new formulas involving curvatures which are analogous to those for the gradient Ricci solitons. As an application, we characterize warped product Einstein metrics when the base is locally conformally flat.

2. C. He, P. Petersen, & W. Wylie, *Warped product Einstein metrics over spaces with constant scalar curvature.* arXiv:1012.3446v1

Abstract: In this paper we study warped product Einstein metrics over spaces with constant scalar curvature. We call such a manifold rigid if the universal cover of the base is Einstein or is isometric to a product of Einstein manifolds. When the base is three dimensional and the dimension of the fiber is greater than one we show that the space is always rigid. We also exhibit examples of solvable four dimensional Lie groups that can be used as the base space of non-rigid warped product Einstein metrics showing that the result is not true in dimension greater than three. We also give some further natural curvature conditions that characterize the rigid examples in higher dimensions.

3. C. He, P. Petersen, & W. Wylie, *The space of virtual solutions to the warped product Einstein equation.* In preparation.

Abstract: We introduce a vector space of virtual warping functions that yield Einstein metrics over a fixed base. There is a natural quadratic form on this space and we study how this form interacts with the geometry of the base. We show that this vector space induces a natural totally geodesic stratification of the underlying manifold and study this stratification.

Published:

4. P. Petersen and W. Wylie, *On the classification of gradient Ricci solitons.* *Geom. & Top.*, 14, pp. 2277-2300, 2010..

Abstract: We show that the only shrinking gradient solitons with vanishing Weyl tensor and Ricci tensor satisfying a weak integral condition are quotients of the standard ones S^n, $S^{n-1}\times\mathbb{R}$, and $\mathbb{R}^n$. This gives a new proof of the Hamilton-Ivey-Perel'man classification of 3-dimensional shrinking gradient solitons. We also show that gradient solitons with constant scalar curvature and suitably decaying Weyl tensor when noncompact are quotients of $\mathbb{H}^n$, $\mathbb{H}^{n-1}\times\mathbb{R}$, $\mathbb{R}^n$, $S^{n-1}\times\mathbb{R}$, or S^n.

5. G. Wei and W. Wylie, *Comparison Geometry for the Bakry-Emery Tensor.*
J. of Diff Geom, 83(2), pp. 377-406, 2009.

Abstract: For Riemannian manifolds with a measure $(M, g, e^{-f}dvol_g)$ we prove mean curvature and volume comparison results when the ∞-Bakry-Emery Ricci tensor is bounded from below and f or $|\nabla f|$ is bounded, generalizing the classical ones (i.e. when f is constant). This leads to extensions of many theorems for Ricci curvature bounded below to the Bakry-Emery Ricci tensor. In particular, we give extensions of all of the major comparison theorems when f is bounded. Simple examples show the bound on f is necessary for these results.

6. P. Petersen and W. Wylie, *On gradient Ricci solitons with symmetry.*
Proc. Amer. Math. Soc., 137, pp. 2085-2092, 2009.

Abstract: We study gradient Ricci solitons with maximal symmetry. First we show that there are no non-trivial homogeneous gradient Ricci solitons. Thus the most symmetry one can expect is an isometric cohomogeneity one group action. Many examples of cohomogeneity one gradient solitons have been constructed. However, we apply the main result in our earlier work to show that there are no noncompact cohomogeneity one shrinking gradient solitons with nonnegative curvature.

7. P. Petersen and W. Wylie, *Rigidity of gradient Ricci solitons.*
Pacific J. of Math., 241(2), pp. 329-345, 2009.

Abstract: We define a gradient Ricci soliton to be rigid if it is a flat bundle $N \times_\Gamma \mathbb{R}^k$ where N is Einstein. It is known that not all gradient solitons are rigid. Here we offer several natural conditions on the curvature that characterize rigid gradient solitons. Other related results on rigidity of Ricci solitons are also explained in the last section.

8. W. Wylie, *Complete shrinking Ricci Solitons have finite fundamential group.*
Proc. Amer. Math. Soc., 136(5) pp. 1803–1806, 2008.

Abstract: We show that if a complete Riemannian manifold supports a vector field such that the Ricci tensor plus the Lie derivative of the metric with respect to the vector field has a positive lower bound, then the fundamental group is finite. In particular, it follows that complete shrinking Ricci solitons and complete smooth metric measure spaces with a positive lower bound on the Bakry-Emery tensor have finite fundamental group. The method of proof is to generalize arguments of Garcia-Rio and Fernandez-Lopez in the compact case.

9. G. Wei and W. Wylie, *Comparison Geometry for smooth metric measure spaces.*
Proc. of the 4th ICCM. Hangzhou, China, Vol. II pp. 191-202, 2007.

Abstract: For smooth metric measure spaces the Bakry-Emery Ricci tensor is a natural generalization of the classical Ricci tensor. It occurs naturally in the study of diffusion processes, Ricci flow, the Sobolev inequality, and conformal geometry. Recent developments show that many topological and geometric results for Ricci curvature can be extended to the Bakry-Emery Ricci tensor. In this article we survey some of these results.

10. W. Wylie, *Noncompact manifolds with nonnegative Ricci curvature.* *J. Geom. Anal.* 16(3), pp. 535-550, 2006.

Abstract: Let (M, d) be a metric space. For $0 < r < R$, let $G(p, r, R)$ be the group obtained by considering all loops based at a point $p \in M$ whose image is contained in the closed ball of radius r and identifying two loops if there is a homotopy between them that is contained in the open ball of radius R. In this article we study the asymptotic behavior of the $G(p, r, R)$ groups of complete open manifolds of nonnegative Ricci curvature. We also find relationships between the $G(p, r, R)$ groups and tangent cones at infinity of a metric space and show that any tangent cone at infinity of a complete open manifold of nonnegative Ricci curvature and small linear diameter growth is its own universal cover.