Math 601 Spring 2008
Take Home Final

Bring the exam to my office by 5pm on Friday May 2nd.
If I am not there, slide it under the door.

You can use Hatcher and your class notes as (the only) reference.

(1) Sow that every continuous map $f\colon \mathbb{S}^2 \to \mathbb{S}^1$ is homotopic to a constant map.

(2) Compute the cohomology groups and ring structure of the Δ-complex obtained by identifying opposite edges (i.e. having no vertices in common) of the standard 3-simplex (so there will be three 1-simplices).

(3) Show that every map $\mathbb{S}^{n+m} \to \mathbb{S}^n \times \mathbb{S}^m$ induces the trivial map in homology.

(4) Show that $\mathbb{S}^2 \times \mathbb{HP}^n$ and $\mathbb{CP}^{2n+1}$ have the same cohomology groups but a different ring structure.

(5) If M and N are two topological manifolds, show that $M \times N$ is orientable iff M and N are both orientable.

(6) Let M_g be an orientable compact surface of genus. Show that there exists a degree 1 map $M_g \to M_h$ iff $g \geq h$.

(7) Consider the space X obtained by gluing a 5-disk $(D^5, \partial D^5)$ onto a 4-sphere S^4 with a gluing map $f : \partial D \to S^4$ of degree 10. Compute the homology groups $H_k(X, \mathbb{Z})$ and $H_k(X, \mathbb{Z}_p)$ for any prime p.

(8) Let M be an 2n-dimensional compact orientable topological manifold. Show that if $H_{n-1}(M, \mathbb{Z})$ has no torsion, then $H_n(M, \mathbb{Z})$ has no torsion either.