

Math 601 Spring 2008  
Homework 10

Due: Friday April 18 in Martin Kerin's mailbox.

A portion of the homework will be graded and returned to you.

- (1) Show that a compact orientable manifold of dimension  $4n + 2$  has even Euler characteristic. What can you say for other dimensions?
- (2) Let  $D$  act properly discontinuously on a topological oriented manifold  $M$ . Define what it means for the group action to be orientation preserving, and show that the quotient is orientable. Indicate how the orientation homomorphism  $O: \pi(M) \rightarrow \mathbb{Z}_2$  we discussed earlier for differentiable manifolds, also works for topological manifolds.
- (3) Let  $M$  be a manifold with boundary  $\partial M$ . Show that an orientation on  $M$  naturally induces an orientation on  $\partial M$ . Show that the boundary homomorphism of the pair  $H_n(M, \partial M) \rightarrow H_{n-1}(\partial M)$  takes fundamental class to fundamental class.
- (4) Let  $M$  be a manifold with boundary  $\partial M$  and define the "double" of  $M$  as  $X = M \cup M$  with the boundary identified. Show that  $X$  is a topological manifold. Show that the Euler characteristic of  $\partial M$  is even. Show that  $\mathbb{C}\mathbb{P}^2$  cannot be the boundary of a 5 dimensional manifold.