Math 601 Spring 2008
Homework 2

Due: Friday February 1 in Martin Kerin's mailbox.

A portion of the homework will be graded and returned to you.

Redo any of the problems in Homework 1 you did not get to

(1) Assume that $f\colon X \to Z$, $g\colon X \to Y$ and $h\colon Y \to Z$ are continuous maps such that $f = h \circ g$. Show that if any two of these are covers, so is the third. What are the precise assumptions under which this is true?

(2) Compute the fundamental group of:
- $\mathbb{R}^n$ minus a point.
- $\mathbb{R}^n$ minus a line.
- $\mathbb{R}^n$ minus a circle.
- $\mathbb{R}^n$ minus a line and a circle. There are four possible spaces. Show they are not homotopy equivalent.

(3) Relate the fundamental group of two n-dimensional manifolds M and N to that of the connected sum $M\sharp N$.

(4) Compute the fundamental group of an orientable surface M_g of genus g and show that M_g is not homotopy equivalent to M_h if $g \neq h$.

(5) If $\sigma\colon E \to G$ is a covering with G a Lie group, show that E is a Lie group also, the multiplication is unique if we choose an identity element over the identity element in G, and σ is a homomorphism.