

Math 601 Spring 2008
Homework 3

Due: Friday February 8 in Martin Kerin's mailbox.

A portion of the homework will be graded and returned to you.

- (1) If $\sigma: E \rightarrow B$ is a covering with deck group D , show that D operates properly and freely on E .
- (2) If D acts properly and freely on E , show that $\sigma: E \rightarrow E/D$ is a covering with deck group D .
- (3) If $E \rightarrow B$ is a covering with B a manifold, show that E is a manifold also. (I relieve you of the duty to show that E is also second countable, which is non-trivial). Also show that the deck group D acts smoothly on E .
- (4) Find an example of a (non-trivial) covering where the deck group is trivial.
- (5) Let G be a connected, simply connected Lie group and Γ a discrete subgroup of the center of G . Show that $G \rightarrow G/\Gamma$ is a covering. Show that every connected Lie group arises from this construction. Find all connected Lie groups whose universal cover is S^3 respectively $S^3 \times S^3$.
- (6) (*Open ended question*) Is every surjective map which has the unique path and homotopy lifting property a covering (you can assume your spaces are sufficiently nice).