

Math 601 Spring 2008
Homework 4

Due: Friday February 15 in Martin Kerin's mailbox.

A portion of the homework will be graded and returned to you.

Homology will always mean singular homology, unless explicitly stated that we are talking about simplicial homology.

- (1) Compute the simplicial homology of the Klein bottle and \mathbb{S}^n for an appropriate Δ complex.
- (2) Let X be obtained by starting with two 2-simplices, $[v_0, v_1, v_2]$ and $[w_0, w_1, w_2]$ and identifying corresponding vertices :

$$X = [v_0, v_1, v_2] \sqcup [w_0, w_1, w_2] / \{v_i \sim w_i\}.$$

What are the simplicial homology groups of X ?

- (3) Compute the simplicial homology of the Δ complex obtained from one simplex by identifying all faces of the same dimension.
- (4) Show that the inclusion $A \rightarrow X$ induces an isomorphism in homology iff $H_*(X, A) = 0$.
- (5) Find an example where $H_*(X, A)$ is different from $H_*(X/A)$.
- (6) (Don't know how difficult this is. Please no silly trivial examples....)
Find a chain complex whose homology is 0. Find two chain complexes A, B with a chain map $f: A \rightarrow B$ which induce the same map in homology, but are not chain homotopic.