Math 601 Spring 2008
Homework 5

Due: Friday February 22 in Martin Kerin's mailbox.

A portion of the homework will be graded and returned to you.

(1) Let Y ba a finite simplicial complex and $X \to Y$ an n-fold cover. Show that the Euler characteristic of X is n times that of Y. Illustrate with the classification of orientable and non-orientable compact surfaces.

(2) What does $H_0(X, A)$ measure?

(3) Show that $\tilde{H}_n(X)$ is isomorphic to $\tilde{H}_{n+1}(SX)$ where SX is the suspension of X, obtained from $X \times I$ by identifying $X \times \{0\}$ to a point and $X \times \{1\}$ to another point.

(4) Let X be a topological manifold (i.e. locally homeomorphic to $\mathbb{R}^n$). Relate, as much as you can, the homology of $X - p$ for $p \in X$ with the homology of X. In case any one wonders, you can assume X is connected.

(5) Let $A \subset X$ and $B \subset Y$ and $f\colon (X, A) \to (Y, B)$ be a continous map of pairs such that $f\colon X \to Y$ and the restriction $f\colon A \to B$ are homotopy equivalences. Show that $f_*\colon H_*(X, A) \to H_*(Y, B))$ is an isomorphism. Is it necessarily true that f is a homotopy equivalence of pairs?

(6) Assume that X is a compact, connected topological space with $\pi_1(X)$ equal to the symmetric group S_3, $H_2(X, \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}_6$, and $H_3(X, \mathbb{Z}) = \mathbb{Z}^2 \oplus \mathbb{Z}_7$. Compute $H_*(X, \mathbb{Z}_2)$ and $H_*(X, Z_{14})$ for $* \leq 3$.