Math 601 Spring 2008
Homework 6

Due: Friday February 29 in Martin Kerin's mailbox.

A portion of the homework will be graded and returned to you.

(1) Let $\mathbb{HP}^n$ be obtained from $\mathbb{H}^{n+1}$ by identifying $v \in \mathbb{H}^{n+1}$ with qv for any nonzero quaternion q. Make $\mathbb{HP}^n$ into a CW complex and compute its homology.

(2) Let X be the space obtained by attaching a ball B^{n+1} to $\mathbb{RP}^n$ where the attaching map $f : \partial B \to \mathbb{RP}^n$ is a composition of a map of degree d from ∂B to the n-sphere $\mathbb{S}^n$ with the standard double covering from $\mathbb{S}^n$ to $\mathbb{RP}^n$. Compute the homology groups of X with coefficients in both $\mathbb{Z}$ and $\mathbb{Z}_2$.

(3) Fix a great k-sphere in $\mathbb{S}^n$. Let X be the quotient of $\mathbb{S}^n$ obtained by identifying antipodal points in this great k-sphere. Compute the homology of X.

(4) Let $M \sharp N$ be the connected sum of two n-dimensional manifolds M and N. Relate as much of the homology of $M \sharp N$ as you can, to the homology of M and N.

(5) If X is a CW complex with k-skeleton's X_k, relate the fundamental group of X with that of X_1, X_2 and X_3. You may assume that X^0 is a point. What is π_1 of $\mathbb{R}P^\infty$ and $\mathbb{C}P^\infty$?

(6) Let $f\colon X \to Y$ be a map such that $f_*\colon H_i(X, \mathbb{Z}) \to H_i(Y, \mathbb{Z})$ is an isomorphism for all i. Show that for any abelian group G, $f_*\colon H_i(X, G) \to H_i(Y, G)$ is an isomorphism also. For the converse, if I know this is true for $G = \mathbb{Z}_p$ for all primes p and for $G = \mathbb{Q}$, does it imply that it is true for $G = \mathbb{Z}$?