Math 601 Spring 2008
Homework 7

Due: Friday March 7 in Martin Kerin's mailbox.

A portion of the homework will be graded and returned to you.

(1) If $X \to Y$ is an n-fold cover, show that the Euler characteristics satisfy $\chi(M) = n\chi(N)$. Do the Betti numbers $(i > 0)$ multiply by n also?

(2) Show that the Euler characteristic of a topological space is the same no matter what field $\mathbb{Z}_p$ or $\mathbb{Q}$ I choose as coefficient group of the homology.

(3) Show that the Euler characteristic of $X \times Y$ is the product of the Euler characteristics of X and of Y.

In (1) – (3) you may assume that they are finite simplicial or finite CW complexes.

(4) Let M^n be a differentiable manifold. Construct a homomorphism $O\colon \pi_1(M) \to \mathbb{Z}_2$ by assigning a loop γ the value 0 or 1, depending whether $\gamma^*(TM)$ is a trivial vectorbundle or not. Then show that M is orientable iff O is trivial. Use this to give another proof that every non-orientable manifold has a 2-fold orientable cover. Discuss its uniqueness and the relationship of all double covers of M. Can you find any conditions on $H_1(M, \mathbb{Z})$ that ensure orientability of M?

(5) Compute the homology of $K \times \mathbb{R}P^3$ with $\mathbb{Z}$ and $\mathbb{Z}_2$ coefficients, where K is the Klein bottle.

(6) Construct a space whose homology is 0 but which is not contractable.