Math 601 Spring 2008
Homework 8

Due: Friday March 21 in Martin Kerin's mailbox.

A portion of the homework will be graded and returned to you.

(1) Let $f\colon X \to Y$ be a map such that $f_*\colon H_i(X,\mathbb{Z}) \to H_i(Y,\mathbb{Z})$ is an isomorphism for all i. Show that for any abelian group G, $f_*\colon H_i(X,G) \to H_i(Y,G)$ is an isomorphism also. For the converse, if I know this is true for $G=\mathbb{Z}_p$ for all primes p, does it imply that it is true for $G=\mathbb{Z}$?

(2) Show that the Klein bottle K and $\mathbb{R}P^2 \vee S^1$ have the same cohomology ring with $\mathbb{Z}$ coefficients, but not with $\mathbb{Z}_2$ coefficients.

(3) Show that $H^n(\mathbb{S}^n, G) = G$ and that a map $f\colon \mathbb{S}^n \to \mathbb{S}^n$ of degree d induces $g \to dg = g + \cdots + g$ in H^n.

(4) Compute the cohomology of a general 3-dimensional lens space $\mathbb{S}^3/\mathbb{Z}_n$ with coefficients in $\mathbb{Z}$ and $\mathbb{Z}_p$.

(5) Compute the cohomology groups of the Klein bottle from its usual Δ-complex structure both with $\mathbb{Z}$ coefficients and with $\mathbb{Z}_2$ coefficients. You computed the homology groups before, but do not use the universal coefficient theorem. Using explicit generators of the cohomology groups, determine the cup product structure of the Klein bottle both with $\mathbb{Z}$ coefficients and with $\mathbb{Z}_2$ coefficients.