

(1) The bigger of the two doesn't exceed 3, so we could have $(1/2/3, 1/2/3)$, the chance is $\frac{9}{36} = \frac{1}{4}$.

(2) Choose one pair from three pairs, and the single one can be chosen from the 4 singles left. $C_3^1 C_4^1 / C_6^3 = \frac{3}{5}$.

$$(3) P_6^3 / 6^3 = \frac{5}{9}.$$

$$(4) P(A \cap B^c \cap C^c) + P(A^c \cap B \cap C^c) + P(A^c \cap B^c \cap C) = \frac{1}{2} \frac{2}{3} \frac{4}{5} + \frac{1}{2} \frac{1}{3} \frac{4}{5} + \frac{1}{2} \frac{2}{3} \frac{1}{5} = \frac{7}{15}.$$

(5) A land two heads and B zero or one: $(\frac{2}{3})^2 \times (1 - (\frac{1}{3})^2)$. A land one head and B land zero: $2 \frac{2}{3} \frac{1}{3} (\frac{2}{3})^2$. Sum up to $\frac{48}{81}$.

$$(6) \text{Two red one green or three red: } (C_6^2 C_4^1 + C_6^3) / C_{10}^3 = 2/3.$$

(7) I set the question to "sum to 8". You could have $(1,1,6)(1,2,5)(1,3,4)(1,4,3)(1,5,2)(1,6,1)$, conditional probability for A to be 1, $6/21$;

$(2,1,5)(2,2,4)(2,3,3)(2,4,2)(2,5,1)$, conditional probability $5/21$;

$(3,1,4)(3,2,3)(3,3,2)(3,4,1)$, $4/21$

$(4,1,3)(4,2,2)(4,3,1)$, $3/21$

$(5,2,1)(5,1,2)$, $2/21$

$(6,1,1)$ $1/21$

$$\text{expectation } E = \frac{6}{21} + 2 \times \frac{5}{21} + 3 \times \frac{4}{21} + 4 \times \frac{3}{21} + 5 \times \frac{2}{21} + 6 \times \frac{1}{21} = \frac{8}{3}.$$

$$(8) P(A \cap B) = P(B)P(A|B) = 1/12. P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/2 + 1/3 - 1/12 = 9/12. P((A \cup B)^c) = 1/4.$$

(9) In this problem you will get the same answer either interpreting "two heads and a tail" as a fixed order "HHT" or three cases ("HHT", "HTH", "THH"), because from the former one to get the later one you will times a factor C_3^2 on both numerators and denominators.

I will do the second way of interpretation.

$$\text{A: } C_3^2 (\frac{1}{4})^2 \frac{3}{4}.$$

$$\text{B: } C_3^2 (\frac{1}{2})^2 \frac{1}{2}.$$

$$\text{C: } C_3^2 (\frac{3}{4})^2 \frac{1}{4}.$$

The conditional probability

$$P(A|condition) = \frac{\frac{1}{3} \times C_3^2 (\frac{1}{4})^2 \frac{3}{4}}{\frac{1}{3} \times (C_3^2 (\frac{1}{4})^2 \frac{3}{4} + C_3^2 (\frac{1}{2})^2 \frac{1}{2} + C_3^2 (\frac{3}{4})^2 \frac{1}{4})} = \frac{3}{20}.$$

$$P(B|condition) = \frac{8}{20}.$$

$$P(C|condition) = \frac{9}{20}.$$

C is the most probable coin to produce such pattern, with 45% chance of being chosen.

$$(10) P_3^3 P_2^2 P_1^1 / P_6^6 = 1/60.$$

$$(11) C_3^2 \left(\frac{3}{5}\right)^2 \frac{2}{5} + \left(\frac{3}{5}\right)^3 = 81/125.$$

(12)

$$\int_1^2 \int_{\sqrt{y-1}}^1 e^{x^3} dx dy = \int_0^1 \int_1^{x^2+1} dy e^{x^3} dx = \frac{1}{3} e^{x^3} \Big|_0^1 = \frac{1}{3}(e-1).$$

(13) Denote $f(x, y) := d^2 = x^2 + (y-1)^2$. $g(x, y) := 4x^2 + y^2 - 4$. The gradient $\nabla f = (2x, 2(y-1))$, $\nabla g = (8x, 2y)$. We have $(x, y-1) = \lambda(4x, y)$ and $g = 0$. Possible extreme value points: $(\pm\frac{\sqrt{5}}{3}, \frac{4}{3})$ when have minimal value $d = \sqrt{2/3}$. And $(0, -2)$ when have max value $d = 3$. $(0, 2)$ is not an extreme value point.