Review Exercises from Math 104 – Solutions

1. Area $= \int_0^1 (x - x^2) = \frac{1}{6}$

2. Using washers, Volume $= \pi \int_0^1 (x^2 - x^4)dx = \frac{2\pi}{15}$.

   Using shells, Volume $= 2\pi \int_0^1 y(\sqrt{y} - y)dy = \frac{2\pi}{15}$.

3. $f'(t) = e^t \cos(e^t) + e^{\sin t} \cos t$

4. $\int_0^{\ln 3} \frac{e^x}{e^x+1} dx = \ln 2$. (Use u-substition with $u = e^x$ or $u = e^x + 1$. If you got $\ln 3$ or $\ln 4$, check your limits of integration again when you change variables.)

5. $\int x \cos(5x)dx = \frac{1}{5} x \sin(5x) + \frac{1}{25} \cos(5x) + C$. (Use integration by parts with $u = x$ and $dv = \cos(5x)dx$.)

6. arc length $= \ln(\sqrt{2}+1)$. Here are the details: Since $y' = (\sec x \tan x)/\sec x = \tan x$, and since $1 + \tan^2 x = \sec^2 x$,

   $$\text{arc length} = \int_0^{\pi/4} \sqrt{1 + (y')^2}dx = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}dx = \int_0^{\pi/4} |\sec x|dx.$$

   Next, note that $\cos x > 0$ on for $0 \leq x \leq \frac{\pi}{4}$, so $\sec x > 0$ as well on this interval; hence $|\sec x| = \sec x$, and we have

   $$\text{arc length} = \int_0^{\pi/4} \sec x dx.$$

   Integrating $\sec x$ requires a trick: Multiply by $\sec x$ by $\dfrac{\sec x + \tan x}{\sec x + \tan x}$ then do a u-substitution with $u = \sec x + \tan x$ to obtain

   $$\int_0^{\pi/4} \sec x dx = \int_1^{\sqrt{2}+1} \frac{du}{u} = \ln u|_1^{\sqrt{2}+1} = \ln(\sqrt{2} + 1).$$

7. The formula for the parameterized line is $\ell(t) = (x(1), y(1)) + t(x'(1), y'(1))$, which after computing $x(1), y(1), x'(1)$, and $y'(1)$ gives us $\ell(t) = (e, 1) + t(\frac{e}{2}, -1)$, or $\ell(t) = (e + \frac{e}{2}t, 1 - t)$. To write this without the parameter t, note that $\frac{dy}{dx} = \frac{y'}{x'}$, which equals $\frac{-1}{e/2} = -\frac{2}{e}$ at $t = 1$. Hence the point-slope equation for a line gives

   $$y - 1 = \left(-\frac{2}{e}\right)(x - e),$$

   which simplifies to $y = -\frac{2}{e}x + 3$.

8. Both series converge. The first converges by comparing it to the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and noting that this new series converges by the p-test. The second series converges by the alternating series test, since $\frac{n^3}{3^n}$ is decreasing in n and goes to 0 as n goes to infinity.