

Additional Exercises

November 18, 2011

Additional questions:

1. Can you find a totally real cyclic $\mathbb{Z}/4\mathbb{Z}$ extension over \mathbb{Q} ? A totally complex one? How about a totally real/complex cyclic degree $2n$ extension where n is arbitrary, can you construct cheap examples?
2. Can you find a cyclic degree 4 extension over \mathbb{Q} that contains $\mathbb{Q}(i)$ as a subfield?
3. Can you find a Galois extension of even order over \mathbb{Q} with absolute value of the discriminant being 23?
4. Someone was looking for a Galois extension of degree 5 over \mathbb{Q} , with the absolute value of the discriminant only divisible by 7. What can you say about the discriminant?
5. Can you find a set of representatives for $\mathbb{Q}_2(i)^*/\mathbb{Q}_2(i)^{*2}$? How about $\mathbb{Q}_2(\zeta_3)^*/\mathbb{Q}_2(\zeta_3)^{*2}$? Find the inner product structure of these spaces w.r.t. the Hilbert symbol, perhaps with the help of PARI. It is interesting! In particular, are these two inner product spaces congruent to each other?