

Answers to Additional Exercises

November 19, 2011

1. see my blog post ([click](#)).

2. No. If the extension is Galois over \mathbb{Q} , you want to take the square root of an element $a + bi$ with $a, b \neq 0 \in \mathbb{Z}$ and $\frac{a+bi}{a-bi} \in \mathbb{Q}^{*2}$. (Why?) In other words, $a^2 + b^2 = c^2$ with $c \in \mathbb{Z}$. But this means the norm of the ideal $(a + bi)$ is a square in \mathbb{Z} . So either $(a + bi)$ contains even power of prime ideal factors that above a prime $(4k + 1)$ that splits, or it contains primes that are inert. It is a nice but elementary exercise to see that $a^2 + b^2$ doesn't have prime factors $4k + 3$, thus $(a + bi)$ is an even power of prime ideals. Since the class number of $\mathbb{Q}(i)$ is 1, it is a square up to a unit. Now adjoining square root of i won't do much good, since it gives a biquadratic extension.

*Can you generalize this phenomenon to the geometric situation?

3. No. The sign of the discriminant is $(-1)^{r_2}$, which is either 0 or even. By Stickelberger, it is 1 or 0 mod 4, contradiction.