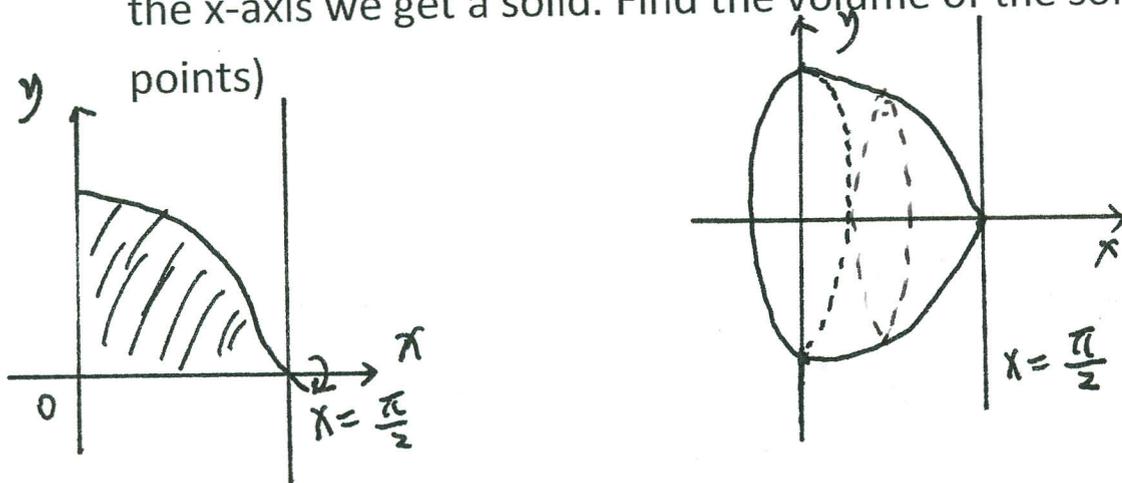


12. We have a region enclosed by the curve $y = \cos x$, the lines $x = 0$, $x = \frac{\pi}{2}$ and $y = 0$. Rotate the region about the x-axis we get a solid. Find the volume of the solid. (10 points)



Cross-section method

x is the variable

$$A(x) = \pi r^2 = \pi \cos^2 x$$

$$a = 0, \quad b = \frac{\pi}{2}$$

$$V = \int_0^{\frac{\pi}{2}} A(x) dx$$

$$= \int_0^{\frac{\pi}{2}} \pi \cos^2 x dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \cos^2 x dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2x}{2} dx$$

$$= \pi \left[\frac{x}{2} \right]_0^{\frac{\pi}{2}} + \frac{\pi}{2} \cdot \left[\frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}}$$

$$= \pi \frac{\pi}{4} + 0 = \boxed{\frac{\pi^2}{4}}$$