

10. $\int \tan^3 x \sec^2 x dx$ (10 points)

Notice that the power of $(\tan x)$ is odd

$$\begin{aligned} & \int \tan^3 x \sec^2 x dx \\ &= \int \tan^2 x \sec^2 x \tan x \sec x dx \\ &= \int (\sec^2 x - 1) \sec^2 x \tan x \sec x dx \end{aligned}$$

Let $u = \sec x$

$$du = \tan x \sec x dx$$

so ~~$\int \tan^3 x \sec^2 x dx$~~

$$\int (\sec^2 x - 1) \sec^2 x \tan x \sec x dx$$

$$= \int (u^2 - 1) u^2 du$$

$$= \int (u^3 - u) du$$

$$= \frac{u^4}{4} - \frac{u^2}{2} + C$$

~~$\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$~~

$$= \frac{\sec^4 x}{4} - \frac{\sec^2 x}{2} + C$$

or $u = \tan x$

$$du = \sec^2 x dx$$

$$\int \tan^3 x \sec^2 x dx$$

$$= \int u^3 du$$

$$= \frac{u^4}{4} + C$$

$$= \frac{\tan^4 x}{4} + C$$