

7. Find the area of the region enclosed by the curve

$y = x^2 - x^3$ and the x-axis. (Hint: To draw the picture, you may need to take derivative to find out where the function is increasing and where it is decreasing.) (10 points)

$$f(x) = x^2 - x^3$$

$$f'(x) = 2x - 3x^2 = x(2 - 3x)$$

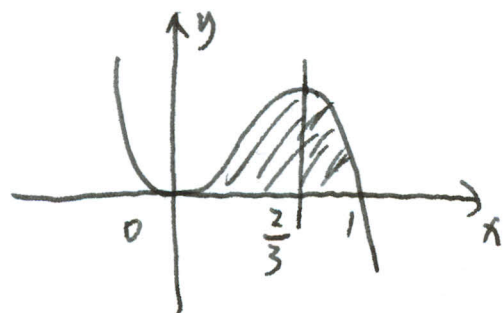
so when $x < 0$ $f'(x) < 0$ $f(x) \downarrow$

when $0 < x < \frac{2}{3}$ $f'(x) > 0$ $f(x) \uparrow$

when $x > \frac{2}{3}$ $f'(x) < 0$ $f(x) \downarrow$

and ~~when~~ we can find the zero-points of $f(x)$: $x^2 - x^3 = 0 \Rightarrow x = 0$ or $x = 1$

so we draw the picture



$$a = 0, \quad b = 1$$

$$A = \int_0^1 (f(x) - 0) dx$$

$$= \int_0^1 (x^2 - x^3) dx$$

$$= \left(\frac{x^3}{3} - \frac{x^4}{4} \right) \Big|_0^1$$

$$= \boxed{\frac{1}{12}}$$