For $2\pi \int_0^1 \frac{x}{1+x^2}\,dx$

$2\pi \int \frac{x}{1+x^2}\,dx$, Let $u = 1+x^2$

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

so $2\pi \int \frac{x}{1+x^2}\,dx = \pi \int \frac{du}{u} = \pi \ln|u| + C$

$= \pi \ln(1+x^2) + C$

so $2\pi \int_0^1 \frac{x}{1+x^2}\,dx = \pi \ln(1+x^2)\Big|_0^1$

$= \pi \ln(1+1) - \pi \ln(1+0)$

$= \pi \ln 2$

For $2\pi \int_0^1 \frac{1}{1+x^2}\,dx$

$\int \frac{1}{1+x^2}\,dx = \arctan x + C$

so $2\pi \int_0^1 \frac{1}{1+x^2}\,dx = 2\pi \arctan x\Big|_0^1$

$= 2\pi (\arctan 1 - \arctan 0)$

$= 2\pi \left(\frac{\pi}{4} - 0\right)$

$= \frac{\pi^2}{2}$

so $V = \boxed{\pi \ln 2 + \frac{\pi^2}{2}}$