

$$\text{Let } u = \frac{x}{2}$$

$$du = \frac{dx}{2}$$

$$dx = 2 du$$

$$\text{So } \frac{\pi}{4} \int \frac{1}{\left(\frac{x}{2}\right)^2 + 1} dx$$

$$= \frac{\pi}{4} \int \frac{2 du}{u^2 + 1}$$

$$= \frac{\pi}{2} \int \frac{du}{u^2 + 1}$$

$$= \frac{\pi}{2} \arctan u + C$$

$$= \frac{\pi}{2} \arctan \frac{x}{2} + C$$

$$\text{So } V = \int_0^2 \frac{\pi}{x^2 + 4} dx$$

$$= \frac{\pi}{2} \arctan \frac{x}{2} \Big|_0^2$$

$$= \frac{\pi}{2} (\arctan 1 - \arctan 0)$$

$$= \frac{\pi}{2} \cdot \frac{\pi}{4}$$

$$= \boxed{\frac{\pi^2}{8}}$$