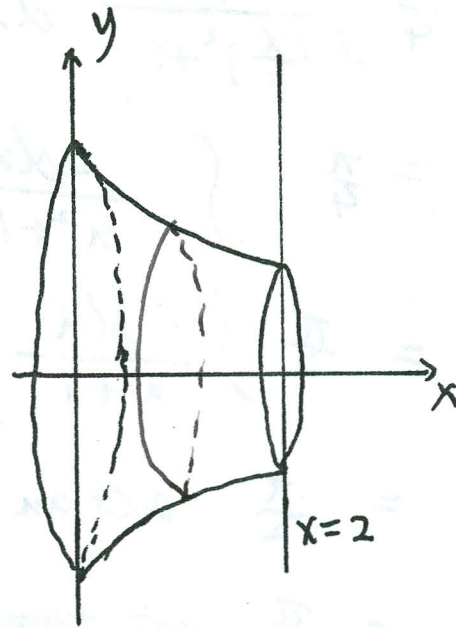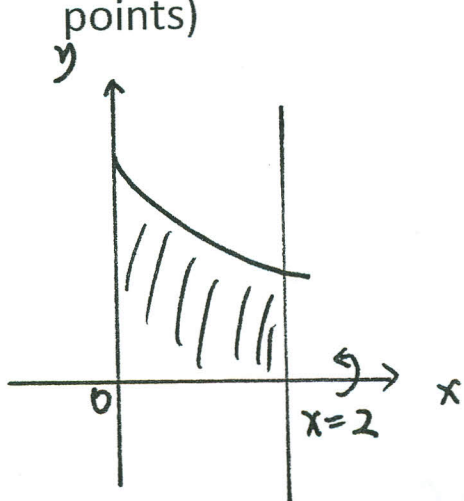5. We have a region enclosed be the curve $y = \frac{1}{\sqrt{x^2+4}}$, the lines $x = 0, x = 2$ and $y = 0$. Rotate the region about the x-axis we get a solid. Find the volume of the solid. (10 points)

cross - section method

x is the variable

$$A(x) = \pi r^2 = \pi \left(\frac{1}{\sqrt{x^2+4}}\right)^2 = \frac{\pi}{x^2+4}$$

$a = 0, \quad b = 2$

so $\quad V = \int_0^2 A(x)\, dx$

$\quad = \int_0^2 \frac{\pi}{x^2+4}\, dx$

We do $\int \frac{\pi}{x^2+4}\, dx$

$$\int \frac{\pi}{x^2+4}\, dx = \frac{\pi}{4} \int \frac{1}{\frac{x^2}{4}+1}\, dx = \frac{\pi}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\, dx$$