

Final Exam

Math 103, Fall 2002

You have 2 hours. There are 200 points total on the test; the point values of each question are listed below. You are allowed one 8.5in x 11in sheet of notes. No calculators, computers, or other aides are permitted. In particular, *turn off your cell phone!* You must show all of your work; **answers without appropriate justification will receive no credit.**

Name: _____

Recitation:

Albert Insogna	Stephan Schoenenberger	Fred Butler	Mahir Can
Tu 8:30	Tu 8:30	M 9:00	M 9:00
Tu 9:30	Tu 9:30	M 10:00	M 10:00
Th 8:30	Th 8:30	W 9:00	W 9:00
Th 9:30	Th 9:30	W 10:00	W 10:00

1. _____ (/24 points)

2. _____ (/24 points)

3. _____ (/22 points)

4. _____ (/30 points)

5. _____ (/20 points)

6. _____ (/30 points)

7. _____ (/25 points)

8. _____ (/25 points)

Total _____ (/200 points)

1. (24 points) Evaluate the following antiderivatives and definite integrals. Do not simplify your answers.

(a) $\int_1^3 \frac{x^4 + 1}{x^2} dx$

(b) $\int_0^{\sqrt{\pi}} x \cos x^2 dx$

(c) $\int (4x - 3)^5 dx$

(d) $\int \frac{x}{x^2 + 1} dx$

2. (24 points) Determine whether the following statements are true (*always* true!) or false (*not* always true). Circle the correct response at right and then *briefly* explain your answer.

(a) If $f(t) > 0$, then $g(x) = \int_0^x f(t) dt$ is positive for $x \geq 0$. **T** **F**

(b) If $f(t) > 0$, then $g(x) = \int_0^x f(t) dt$ is increasing for $x \geq 0$. **T** **F**

(c) If f is a continuous function, then it has an antiderivative. **T** **F**

(d) Given a function $f(x)$, Newton's method converges to a root of the equation $f(x) = 0$ for any initial guess x_0 . **T** **F**

3. (22 points)

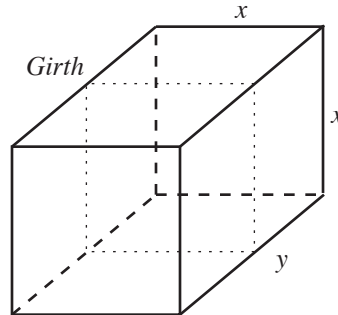
(a) Say that you are given two functions f and g , both of which have the domain $(-\infty, \infty)$. Suppose that $f'(x) = x^2$ and $g'(3) = 17$. What additional piece of information do you need in order to compute $(f \circ g)'(3)$ (i.e. the derivative of f composed with g at $x = 3$)?

(b) Suppose that $h(2) = 4$ and $h'(2) = 1$.

i. Find the the tangent line to the graph of $xh(x)$ at the point $(2, 8)$.

ii. Use your answer in part (i) to estimate the value of $2.1 \cdot h(2.1)$.

4. (30 points) You want to pack up and ship a large number of paperweights in rectangular boxes using the US postal service. Assume that the boxes have a square cross section, as shown in the following diagram:



By definition, the *width* of the box is labeled x and the *length* of the box is labeled y in the diagram. The *girth* of the box is the perimeter of a cross section of the box, which is represented by the dotted square in the diagram.

You find out from the postal service that the sum of the length and girth of a box can be at most 108 inches.

- (a) Write down a function $V(x)$ that expresses the volume of the box in terms of the width x .
- (b) What widths x make sense for the problem? In other words, what is the domain of $V(x)$?

(Problem is continued on the next page)

- (c) What are the dimensions of the box of maximum volume that can be shipped?
Be sure to justify that your result actually gives the maximum.

5. (20 points) Let $f(x) = x^3 - 3x + 2$.

(a) Where are the critical points of $f(x)$? What are the y -values of the critical points?

(b) On which intervals is $f(x)$ increasing? On which intervals is it decreasing?

(c) On which intervals is $f(x)$ concave up? On which intervals is it concave down?

(d) Find an interval $[a, b]$ on which the Left Hand Sum is an overestimate for $\int_a^b f(x) dx$.

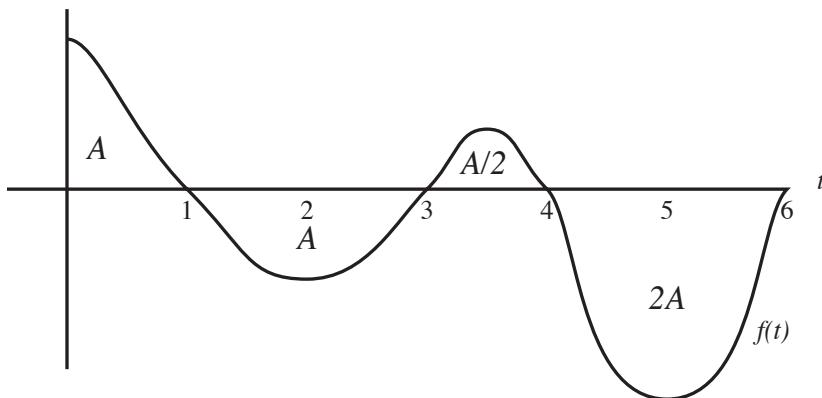
6. (30 points) Suppose that you have the misfortune of being the CEO of a major airline.¹ Though you'd rather lock yourself in your office and not come out until Congress bails you out, you must face the numbers. Your beancounters tell you that the rate of change of your debt (i.e. your deficit) is given by $d'(t) = t + 1$, where $d(t)$ is in millions of dollars and t is in weeks. Here, we're thinking of the debt as a *positive* number, so this means that your debt is getting worse.

(a) If your current debt is \$ 400 million, what will your debt be in 6 weeks?

(b) How long will it take for your debt to reach \$ 460 million?

¹With the exception of Southwest, of course.

7. (25 points) Suppose that $g(x) = \int_0^x f(t) dt$, where the function $f(t)$ is given by the following graph:



Here, the labels A , $A/2$, and $2A$ refer to the areas between the graph of $f(t)$ and the t -axis.

- (a) What is $g(1)$? What is $g(3)$? What is $g(6)$? Your answers should be in terms of A .
- (b) What is $g'(x)$?
- (c) On which intervals is $g(x)$ increasing? On which intervals is $g(x)$ decreasing?
- (d) Where does $g(x)$ have a local maximum? Where does $g(x)$ have a local minimum?
- (e) Where does $g(x)$ have an absolute minimum on the interval $[0, 6]$?

8. (25 points) You have a wine glass whose sides are modeled by rotating the portion of the graph of $y = x^2$ that lies in the first quadrant between $y = 0$ and $y = 5$ around the y -axis. All measurements are in inches.

(a) Sketch the portion of the graph described above. Also include the axis of rotation in your sketch.

(b) Suppose that you pour water into the glass until the water is a height h above the bottom of the bowl (Suppose $0 \leq h \leq 5$). Write down and evaluate an integral that gives you the volume of the water (in terms of h !).

(c) Suppose that you continue to pour water into the glass at a rate of $2 \text{ in}^3/\text{sec}$. What is the rate of change of the height of the water when it reaches the top of the glass? (**Hint:** Think of the height h in part (b) as a function of the time t , i.e. $h = h(t)$)

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