University of Pennsylvania
Mathematics Department

Mathematics 103 Fall 2005 **Make up** Final Exam

Name:______________________ Professor:__________________
TA:_______________________

- There are ten problems. Most have several parts

- You may use one 8.5 by 11 inch "cheat sheet."
  Nothing else. Calculators are not allowed

- Work each problem in the space provided. If that space is insufficient continue on the back of the page.

Please do not write below this line.

Point Totals

Problem 1	______	of 12
Problem 2	______	of 8
Problem 3	______	of 8
Problem 4	______	of 8
Problem 5	______	of 8
Problem 6	______	of 12
Problem 7	______	of 12
Problem 8	______	of 12
Problem 9	______	of 10
Problem 10	______	of 10
Total Score	______	**of 100**

1. Compute the limits :

(*i*) $\lim\limits_{h \to 0} \dfrac{\sqrt{4+h} - \sqrt{4-h}}{2h}$

(*ii*) $\lim\limits_{x \to \infty} \dfrac{\sqrt{16x^4 + 12x^2 - 8}}{2x^2 - 10}$

2. Find the area of the region enclosed by the graphs of $y = 6x - x^2$ and $y = x + 4$

3. A rectangular field adjacent to a river is to be enclosed. Fencing along the river costs \$5 per foot but fencing alng the otheer three sides costs only \$3 per foot. The area of the field is to be 4800 square feet.

(i) What are the dimensions of the rectangular field of minimum cost?

(ii) What is the minimum cost?

4. Two commercial airlines are flying at the same altitude along straight line courses that intersect at right angles. Plane A is approaching the intersection point at 400 miles per hour and plane B is approaching the intersection point at 500 miles per hour. At what rate is the distance between the planes changing when plane A is 6 miles from the intersection point and plane B is 8 miles from the intersection point.

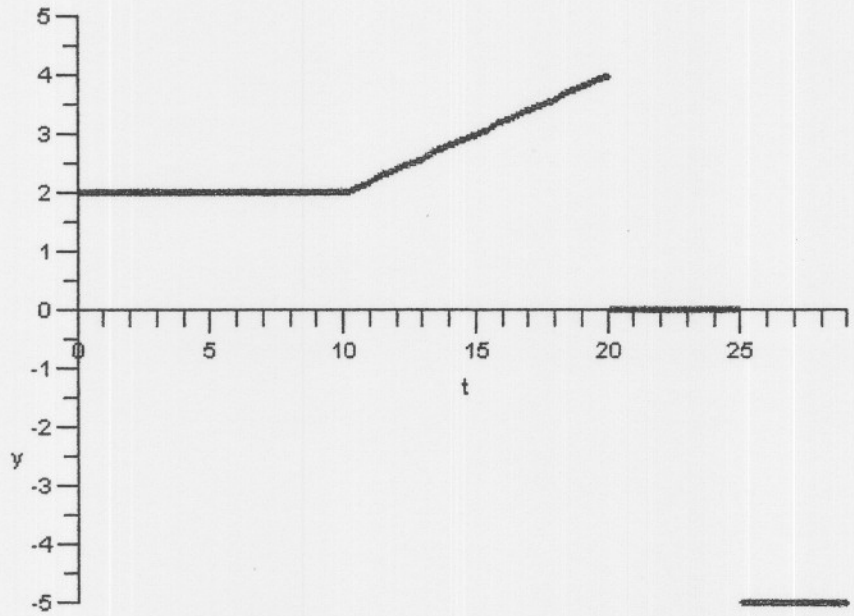
5. Derivatives: Find $\dfrac{dy}{dx} = f'(x)$ if :

(i) $f(x) = \dfrac{\cos x}{\sqrt{x^2+1}}$

(*ii*) $x^3 + x^2y^2 + y^4 = 0$ (implicitly)

(*iii*) $f(x) = \int_0^{x^2} \sin t dt$

6. The following diagram represents the speed of a student walking along Walnut street to his Math103 lecture in DRL. He starts walking at 9.30am and arrives at 9.59am. The time is given in minutes after the student leaves the house and the speed is given in miles per hour.



a) What distance has the student covered after 20min?
b) Compute the total distance the student has walked.
c) What is the distance between the student's home and the math department?
d) Using only very simple computations, show that the student had reached the math department for the first time before 9.45am.

7. Integrals: Evaluate the following integrals:

(*i*) $\int_{-3}^{3}\left(4x^3+7x-4\right)dx$

(*ii*) $\int_{0}^{3}\frac{2x}{\sqrt{16+x^2}}dx$

(iii) $\int_{0}^{4}\left|\sqrt{x}-1\right|dx$

8. For y = $f(x) = \dfrac{x^2+4}{x}$:

(*i*) What is the domain? (*ii*) Find the asymptotes (if any)

(*iii*) Where (on what intervals) is $f(x)$ increasing/decreasing?

Find all local maxima and all local minima

(*iv*) Where (on what intervals) is $f(x)$ concave up/concave down?

(*v*) Use the information you have obtained to draw a graph of $f(x)$. Include the asymptotes , if you have found any.

9. If $f(x)$ is an **even** , continuous function for which $\int_{-2}^{2} f(x)dx = 6$

and if also $\int_{0}^{4} f(x)dx = 10$ then

(*i*) $\int_{-2}^{2} -3f(x)dx =$

(*ii*) $\int_{2}^{4} f(x)dx =$

(*iii*) $\int_{-1}^{3} f(x-1)dx =$

10. Circle True or False for each statement

(*i*) If $f''(3) = 0$, then $f(x)$ can NOT have a local maximum at $x = 3$. **T** **F**

(ii) If $f''(3) > 0$ then $f(x)$ MUST have a local minimum at $x = 3$ **T** **F**

(iii) If $g(x)$ is an odd function and if $g(x)$ has a local maximum at $x = 3$ then $g(x)$ has a local minimum at $x = -3$ **T** **F**

(*iv*) It is possible to have a function $f(x)$ such that **for every** x
(a) $f(x) < 0$, (b) $f'(x) < 0$ and (c) $f''(x) > 0$ **T** **F**

(*v*) If $f(0) = 3$ and $f'(x) > 3$ for every x , then $f(3) > 12$ **T** **F**