University of Pennsylvania
Math 103-Final Exam
Fall 2013

Name _____ **(PRINT)** **Penn ID#** _____

Professor : (circle one) **Rimmer** **Block** **Clee**

Recitation Number_____ **Rec. Day** _____ **Rec. Time**_____

This exam has 10 multiple choice questions and 5 open-ended questions. Each question is worth 10 points. On the multiple part open-ended questions, the point value of each part can be found next to the question. Partial credit will be given for the entire exam so be sure to show all work. On the multiple choice questions, circle the correct answer and give supporting work, a correct answer with little or no supporting work will receive little or no credit. Use the space provided to show all work. A sheet of scrap paper is provided at the end of the exam. If you write on the back of any page please indicate this in some way.
You have 120 minutes to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up continue to stay seated until someone comes by to collect your exam.

Once you have completed the exam, sign the academic integrity statement below.
Do **NOT** write in the grid below. It is for grading purposes only.

Problem	Points		Problem	Points
1			9	
2			10	
3			11	
4			12	
5			13	
6			14	
7			15	
8				
Total			Total	

Final Exam Score

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

_____
Name (printed)

_____
Signature

_____
Date

1. Consider the function

$$f(x)=\begin{cases} 3 & \text{if } x\le -1 \\ 2x+5 & \text{if } -1<x<0 \\ 2 & \text{if } x=0 \\ \dfrac{x^2-8x+15}{3-x} & \text{if } x>0 \text{ and } x\neq 3 \end{cases}.$$

2 points (a) Determine $\lim_{x\to -1} f(x)$, if it exists. Otherwise write DNE.

4 points (b) Determine $\lim_{x\to 0} f(x)$, if it exists. Otherwise write DNE.

4 points (c) Determine $\lim_{x\to 3} f(x)$, if it exists. Otherwise write DNE.

2. Let

$$f(x)=\frac{1}{x^2}$$

Find $f'(x)$ **by using the definition of the derivative**.

3. Suppose $g, h,$ and j are differentiable functions with the values for the function and derivative given in the following table:

x	g(x)	h(x)	j(x)	g'(x)	h'(x)	j'(x)
-1	3	0	1	-1	-2	-2
0	2	3	0	-2	3	-2
1	0	-1	-2	-2	-2	-1

Let

$$f(x)=\frac{h(x)}{j(x)}+g(h(x)).$$

Find $f'(1)$.

(a) $\frac{-5}{4}$ (b) $\frac{5}{4}$ (c) $\frac{7}{2}$ (d) $\frac{11}{4}$ (e) $\frac{5}{2}$ (f) 3

4. Let

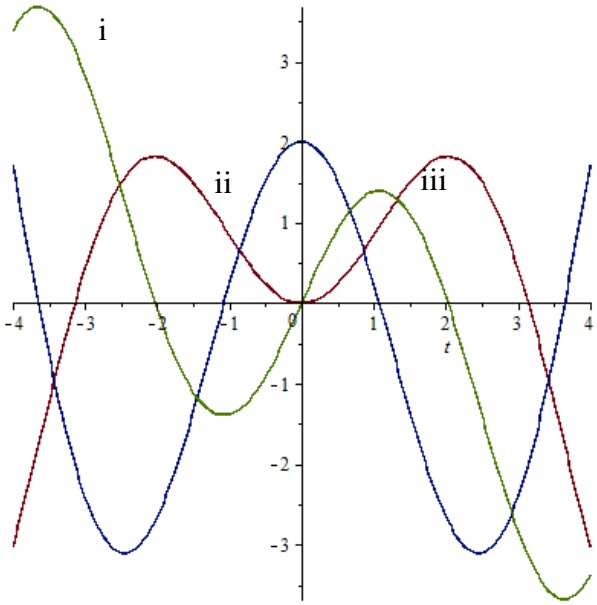
$$y = \arcsin(x)\ .$$

Find the value of its second derivative evaluated at $\dfrac{\sqrt{3}}{2}$

(a) $\dfrac{1}{2}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\sqrt{3}}{2}$ (d) $2\sqrt{3}$ (e) $4\sqrt{3}$ (f) $8\sqrt{3}$

5. Sand falls from a conveyor belt at the rate of 30 cubic feet per minute onto the top of a conical pile. The height of the pile is always equal to the base diameter. At what rate is the height of the pile increasing at the instant when the height is 10 feet?

6. The curves (i), (ii), and (iii) in the graph below are graphs of a function f and its first and second derivatives. Which curve is f, which is f', and which is f''? Explain in order to get full credit.



(a) i) f ii) f' iii) f'' (b) i) f ii) f'' iii) f'

(c) i) f' ii) f iii) f'' (d) i) f' ii) f'' iii) f

(e) i) f'' ii) f iii) f' (f) i) f'' ii) f' iii) f

7. Consider the limits :

$$L=\lim_{x\to 0}\frac{1-\cos x}{x^2},\qquad M=\lim_{x\to 3}\frac{x^2+15}{3x^2-3},\qquad N=\lim_{t\to 0}\left(\cos t\right)^{t^{-2}}$$

Which of the following is true?

(a) $L>M>N$ (b) $L>N>M$ (c) $M>L>N$

(d) $M>N>L$ (e) $N>L>M$ (f) $N>M>L$

8. Let

$$f\left(x\right)=\frac{2x^2-x+3}{x-1}$$

1 point (a) Find the $x-$intercepts and the $y-$intercepts.

1 point (b) Find the asymptotes: vertical, horizontal and slant.

2 points (c) Find the interval(s) where the function is increasing and where the function is decreasing.

2 points (d) Find the interval(s) where the function is concave up and where the function is concave down.

2 points (e) Where does the function have local maxima? Where does the function have local minima?

2 points f) Graph the function.

9. You want to construct a cylindrical container that contains 400 cubic inches of liquid. The material for the top and bottom costs 5 cents per square inch and the material for the sides costs 2 cents per square inch. What should be the dimensions of the container if you want to minimize the cost?

10. Evaluate the limit

$$\lim_{x\to 0}\frac{e^x-1-x-\frac{1}{2}x^2}{x^3}$$

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$ (e) $\frac{1}{6}$ (f) 0

11. Evaluate the integral below.

$$\int_0^{\frac{\pi}{3}}\left(\sin x-\cos x+\sec^2 x\right)dx$$

(a) $\frac{1+\sqrt{3}}{2}$ (b) $\frac{1-\sqrt{3}}{2}+\frac{\sqrt{3}}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $2+2\sqrt{3}$ (e) $\frac{\sqrt{3}}{3}$ (f) $\sqrt{3}$

12. Let $f(x)$ be a function that is continuous for all x. Suppose it is known that

$$\int_{-2}^{3} f(x)\,dx=4 \text{ and } \int_{-2}^{6} f(x)\,dx=9$$

Then calculate

$$\int_3^6\left(2x^2-5f(x)\right)dx$$

(a) 81 (b) 91 (c) 101 (d) 111 (e) 131 (f) 141

13. Let

$$G(x) = \int_2^{\sqrt{x}} t \ln t \, dt$$

Find the derivative of G evaluated at e^8 .

(a) 0 (b) $\frac{1}{2}\sqrt{e}$ (c) $\ln 2$ (d) $\sqrt{2}$ (e) 2 (f) $2\sqrt{2}$

14. Evaluate the integral below.

$$\int_{\ln 2}^{\ln 3} \frac{2e^{2x}}{1+e^{2x}} dx$$

(a) e (b) 1 (c) $\ln 2$ (d) $\ln 3$ (e) 2 (f) e^2

15. Find the area of the region bounded above by the line $y = x - 1$ and bounded below by the parabola $y = x^2 - 2x - 1$.

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) $\frac{5}{2}$ (e) 4 (f) $\frac{9}{2}$

ANSWERS:

1. a) 3 b) 5 c) 2 2. $f'(x)=\frac{-2}{x^3}$ show using the definition

3. D 4. E 5. $\frac{6}{5\pi}$ 6. C 7. D

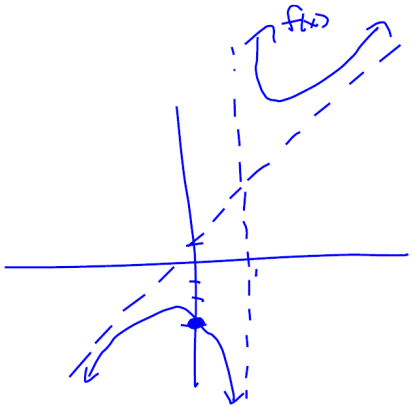
8. a) (0, -3) b) $x=1$ and $y=2x+1$

c) Decreasing $\left(1-\sqrt{2},1\right)\cup\left(1,1+\sqrt{2}\right)$ d) Concave down: $(-\infty,1)$

Increasing $\left(-\infty,1-\sqrt{2}\right)\cup\left(1+\sqrt{2},\infty\right)$ Concave up: $(1,\infty)$

e) local max at $x=1-\sqrt{2}$, local min at $x=1+\sqrt{2}$

f)



9. $r=2\sqrt[3]{\frac{10}{\pi}}$, $h=10\sqrt[3]{\frac{10}{\pi}}$ 10. E 11. A 12. C 13. E

14. C 15. F