Math 202 **Exam 3** Jerry L. Kazdan
December 10, 2013 12:00 — 1:20

Directions: Part A has 6 shorter problems (8 points each) while Part B has 4 traditional problems (13 points each). 100 points total].
To receive full credit your solution should be clear and correct. Neatness counts. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one 3 ×5 with notes on both sides.

Part A: Six shorter problems, 8 points each [total: 48 points]

A-1. Say a function $f(x)$ has the properties $f'(x) = 2\cos 2x$ for all $x \in \mathbb{R}$ and $f(0) = 0$. Show that $f(x) = \sin 2x$. [Hint: To show that "$A = B$", it is often easiest to show that "$A - B = 0$".]

A-2. Find the continuous function f and constant C so that $\displaystyle\int_1^x f(t)\,dt = x\cos(\pi x) + C$.

A-3. Give an example of a bounded continuous function $f(x)$, $x \in \mathbb{R}$, that does not attain its supremum. A clear sketch is adequate.

A-4. Let a_n be a sequence of real numbers that converges to A. If $a_n \ge 0$, give a clear proof that $A \ge 0$.

A-5. Give an example of a sequence, $f_n(x)$, of functions on the interval $[0,\, 1]$ that converge pointwise to 0 but do *not* converge uniformly. A good sketch is adequate.

A-6. Let $p(x) = x^9 + a_8x^8 + \cdots + a_1x + a_0$. Prove (clearly) that $\displaystyle\lim_{x\to-\infty} p(x) = -\infty$.

Part B: Four traditional problems, 13 points each [52 points[

B-1. Let $f(x)$ and $g(x)$ be differentiable for $x \in [a,\, b]$ and let $p \in (a,\, b)$. Show directly from the definition of the derivative that the product, $f(x)g(x)$, is differentiable at the point p and the derivative is given by the usual rule: $(fg)'(p) = f'(p)g(p) + f(p)g'(p)$.

B-2. Let f be a continuous function on the interval $[a,\, b]$. If $\displaystyle\int_a^b f(x)\,dx = 0$, show there is a point $c \in (a,\, b)$ so that $f(c) = 0$.

B-3. Let $I_k = \{x \in \mathbb{R} \mid a_k \le x \le b_k\}$ be closed bounded *nested* intervals, so $I_{k+1} \subseteq I_k$.

a) Use the completeness property of the real numbers ("bounded monotone sequences converge") to show that there is at least one point in the intersection, $\cap I_k$.

b) Give an example where the intersection is the *whole* interval $-1 \le x \le 1$.

B-4. Let $f(x)$ be continuous on the interval $[0, 1]$ and $g_n(x)$ be the sequence of functions in the figure. Show that

$$\lim_{n\to\infty} \int_0^1 f(x) g_n(x)\, dx = f(0).$$

SUGGESTION First do the case where $f(x) \equiv 1$.