Signature | PRINTED NAME

Math 202 | **Exam 3** (Make-up) | Jerry L. Kazdan

January 22, 2014 | 6:00 — 8:00 PM

DIRECTIONS: Part A has 6 shorter problems (8 points each) while Part B has 4 traditional problems (13 points each). 100 points total].
To receive full credit your solution should be clear and correct. Neatness counts. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one 3 ×5 with notes on both sides.

PART A: Six shorter problems, 8 points each [total: 48 points]

Score	
A-1	
A-2	
A-3	
A-4	
A-5	
A-6	
B-1	
B-2	
B-3	
B-4	
Total	

A-1. If two differentiable functions on the interval $(0, 1)$ have the same derivative, prove that they differ by a constant.

A-2. Find the continuous function f and constant C so that $\int_0^x f(t)\,dt = e^{\cos x} + C$.

A-3. Let $\{c_n\}$ be a sequence of (possible complex) numbers. If the series $\sum_{n=1}^{\infty} c_n$ converges, show that $c_n \to 0$.

A-4. Let a_n be a sequence of real numbers that converges to A. If $a_n \le 1$, give a clear proof that $A \le 1$.

A-5. Give an example of a sequence, $f_n(x)$, of continuous functions on the interval $[0, 1]$ that converge pointwise to some function $g(x)$ but do *not* converge uniformly. A good sketch is adequate.

A-6. Give a (clear) proof that $\displaystyle\lim_{t\to\infty} \frac{3t^4 - 2t^3 + 5}{2t^4 + 5} = \frac{3}{2}$

PART B: Four traditional problems, 13 points each [52 points[

B-1. Let $f(x) = x^3$ and P_n be a partition of the interval $[0, 2]$ into n intervals of equal length. Find a number n to insure that the upper Riemann sum $U(f, P_n)$ is within .01 of $\int_0^2 x^3\,dx$.

B-2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function and let a be a positive real number. Show that the function

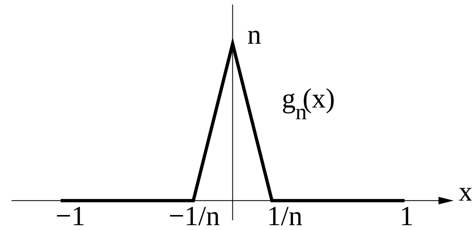
$$G(x) := \int_{-a}^{a} f(x+t)\, dt$$

is differentiable and has a continuous derivative and compute $G'(x)$. [SUGGESTION: Make a preliminary change of variable.]

B-3. Let a_n be a sequence of real numbers that converges to zero, $a_n \to 0$ as n goes to infinity. Show that

$$\lim_{n\to\infty} \frac{a_1 + a_2 + \cdots + a_n}{n} = 0.$$

B-4. Let $f(x)$ be continuous on the interval $[-1, 1]$ and $g_n(x)$ be the sequence of functions in the figure. Show that



$$\lim_{n\to\infty}\int_{-1}^{1} f(x)g_n(x)\,dx = f(0).$$

SUGGESTION First do the case where $f(x) \equiv 1$.