Signature PRINTED NAME

Math 210 **Exam 1** Jerry L. Kazdan
October 9, 2003 10:30 — 11:50

DIRECTIONS: This exam has 7 problems (*10 points each*). To receive full credit your solution must be clear and correct.. Closed book, no calculators – but you may use one 3"x5"card with notes. Please box your answers.

Score	
1	
2	
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Total	

1. (SHORT ANSWER)
   a). If you roll a die 3 times, what is the probability of getting at least one of them showing a 2?

   b). Say you have tossed a (fair) coin 99 times and gotten 83 "heads" and 16 "tails". What is the probability that on the next toss it will shown a "head"?

   c). What day of the week will it be on March 1, 2114? [In case it helps, January 1, 2113 will be a Sunday.]

2. In your office, say your personal code to use the photo copy machine is the last four digits of your employee number.

   If 100 people share that copier, what is the probability that at least two people have the same code?

3. The next three players in a game win 30%, 40% and 25% of the time, respectively. What is the likelihood that *none* of them will win this time? [EQUIVALENT WORDING: It is the fifth inning of a baseball game. The batting averages of the next three batters are .300, .400, and .250. Say they face an average pitcher. What is the likelihood that *none* of them will get a hit this inning?]

4. A large number, N, of people are subjected to a blood test, the result of which is either "positive" or "negative". It can be given in two ways:

   i). Each person can be tested separately, so N tests are required.

   ii). The blood samples of k persons can be pooled and analyzed together. If this test is negative, then one test suffices for the k people, while if the test is positive, each of the k people must be tested separately so $k+1$ tests are required for the k people.

   Assume that the probability, p, that a test is positive is the same for all people and that these events are all independent.

   a). Find the probability that the test for a pooled sample of k people will be positive.

   b). What is the expected value of the number of tests necessary under plan ii)? [Assume that N is divisible by k].

   c). What is the standard deviation under plan ii)?

5. Write a Perl (or Maple) script that uses the "Monte Carlo" method for estimating the area of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} \leq 1.$$

This ellipse lies inside the box $Q = \{(x, y) : |x| \leq 3, |y| \leq 2\}$. The Monte Carlo method says to pick many points at random in the box Q. The area of the ellipse will correspond to the percentage of the points that in the ellipse.

REMARK: In case it helps, here is an *unrelated* perl script.

```
#!/usr/bin/perl -w
#-------------------------- What This Does ----------------------------
# If you toss a coin at random N times, how many heads do you get?
#---------------------------- Main Program ----------------------------
$N = 10000;     # number of tosses
$heads = 0;     # initialize

for ($k=0; $k<$N; $k++) {
    $toss = int(rand(2));
    if ($toss == 0) {$heads = $heads +1;}
    if (($k % 1000) == 0) {print "In $k tosses there were $heads heads.\n";}
}
print "\n   Summary: In $N tosses there were $heads heads.\n";
```

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6. The following describes a web page. How will it appear? (fill-in the blank page below).

```
<html><head><title>Fall Break</title></head>

<body bgcolor=yellow>
<center><H1> Fall Break</H1></center>

Fall Break begins <I>this</I> weekend.

Enjoy it.

<P>
    Bye Bye
</P>
</body></html>
```

7. A friend is about to take a test for a relatively rare cancer that has an incidence of 0.1% among the general population. Thus, before taking the test, and in the absence of any other evidence, the best estimate of the likelihood of her having the cancer is 1 in 1000.

   Extensive trials have shown that the reliability of the test is 98%, that is, it gives a positive result in 2% of the cases where no cancer is present (*false positive*). Moreover, about 5% of the time the test fails to detect the cancer even though it is present (*false negative*).

   a) If your friend tests positive, what is the probability that she has this cancer? [Do not take time to "simplify" your answer.]

   b) If your friend tests negative, what is the probability that she has this cancer? [Again, do not take time to "simplify" your answer.]