

DIRECTIONS: This exam has 8 problems (10 points each). To receive full credit your solution must be clear and correct. Closed book, no calculators – but you may use two 3" × 5" cards with notes. Please box your answers.

1. Spoiled Fruit. Discuss which of the two farms does a better job at minimizing the amount of spoiled fruit.

	Eva's Farm			Susan's Farm		
	total	spoiled	% spoiled	total	spoiled	% spoiled
melons	1,000	100	10%	5,000	70	1.4%
oranges	150,000	1,000	0.67%	10,000	50	0.5%
<b>Summary</b>	151,000	1,100	0.73%	15,000	120	0.8%

[We are seeking a *quantitative* response, not an intuitive one. Since you don't have calculators, approximate calculations will be adequate.]

2. Find a  $3 \times 3$  matrix  $A$  that reflects a point  $\mathbf{P} = (x_1, x_2, x_3)$  in  $\mathbb{R}^3$  across the plane  $x_3 = 0$  and then, in  $(y_1, y_2, y_3)$  space rotates it by 90 degrees about the  $y_2$ -axis.
3. Let  $A$  be a  $3 \times 3$  matrix with eigenvalues  $\lambda_1, \lambda_2, \lambda_3$  and corresponding (independent) eigenvectors  $V_1, V_2, V_3$  which we can therefore use as a basis. Of course  $AV_j = \lambda_j V_j$ .
- a) If  $X = aV_1 + bV_2 + cV_3$ , compute  $AX, A^2X$ , and  $A^{35}X$  in terms of  $\lambda_1, \lambda_2, \lambda_3, V_1, V_2, V_3, a, b$  and  $c$  (only).
- b) If  $\lambda_1 = 1, |\lambda_2| < 1$ , and  $|\lambda_3| < 1$ , compute  $\lim_{k \rightarrow \infty} A^k X$ . Explain your reasoning clearly.
4. You and a friend agree to meet at a restaurant between 12:00 and 1:00 every Wednesday for lunch. Suppose you both arrive between 12:00 and 1:00, but at times chosen at random.
- a) What is the probability distribution function for the amount of time the first to arrive must wait for the other?
- b) What is the probability density?
- c) What is the expected waiting time?
- d) Write a specific formula for the standard deviation of the waiting time [but don't take the time now to evaluate the resulting integral].
5. There are two local branches of the Limousine Rental Company, one at the Airport and one in the City, as well as branches Elsewhere. Say every week of the limousines rented from the Airport 30% are returned to the City and 2% to branches located Elsewhere. Similarly of the limousines rented from the City 30% are returned to Airport and 2% to Elsewhere. Finally, say 10% of the limousines rented from Elsewhere are returned to the Airport and 10% to the City.
- If initially there are a total of 110 limousines, what is their long-term distribution?

Score	
1	
2	
3	
4	
5	
6	
7	
8	
<i>Total</i>	

6. Say you seek a parabola with the special form  $y = a(x - 1)^2 + b$  to pass through the three data points  $(0, 2)$ ,  $(1, 0)$ ,  $(2, 3)$ .
- Write the (over-determined) system of equations you would like to solve ideally.
  - Using the method of least squares write the *normal* equations for the coefficients  $a$ ,  $b$ .
  - Explicitly find the coefficients  $a$  and  $b$ .

7. At the conclusion of the regular (before playoffs) 2002 Ivy League Basketball season there were three teams tied for first place. The standing are in the win-loss table on the left.

Because only one team would qualify for the NCAA tournament there were two playoff games to determine the team that would go to the tournament. An alternative would have been to use the methods discussed in class to rank the teams. In particular we may assume that the final ranking of a team is somehow related to the ranking of the teams that it has defeated during the regular season. The table on the right shows how many times each team beat the other teams. For instance, Penn won over Cornell twice while Cornell never beat Penn.

TEAM	CONF.
Penn	11 - 3
Yale	11 - 3
Princeton	11 - 3
Brown	8 - 6
Harvard	7 - 7
Columbia	4 - 10
Dartmouth	2 - 12
Cornell	2 - 12

	Pe	Y	Pr	B	H	Col	D	Cor
Penn	0	1	2	2	1	1	2	2
Yale	1	0	1	1	2	2	2	2
Princeton	0	1	0	2	2	2	2	2
Brown	0	1	0	0	1	2	2	2
Harvard	1	0	0	1	0	2	2	1
Columbia	1	0	0	0	0	0	1	2
Dartmouth	0	0	0	0	0	1	0	1
Cornell	0	0	0	0	1	0	1	0

- Show how you would use the methods discussed in class to rank these teams. Explicitly do one step (iteration) of the computation so I can understand your procedure.
- I carried out the computation and obtained the following Perron eigenvector:

$$V = (0.238, 0.215, 0.194, 0.119, 0.118, 0.0667, 0.0202, 0.0291)$$

What ranking does this give?

- To save time writing, use:

$$A = \begin{pmatrix} 0 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \\ 1 & 0 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 1 & 0 & 2 & 2 & 2 & 2 & 2 \\ 0 & 1 & 0 & 0 & 1 & 2 & 2 & 2 \\ 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

8. The following was used to produce a form on a web page:

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```
<html><head><title>division</title></head>
<body bgcolor=white>
<center> <h2>Math 210 Using Perl to Divide</h2> </center>
<form action="/cgi-bin/division.pl">
You specify:
<p>
<center><b>x =</b><input type=text name="x" size=15>
<br><b>y = </b><input type=text name="y" size=15</center>
<p>
This computes the division <b>x / y</b>.
<p>
<center><input type=submit value="Submit"></center>
</form>
</body></html>
```

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Your task: write a follow-up perl script that does this division and reports the result as a web page. Your script should *first* check if the denominator is zero. If so, instead of doing the computation it gives an appropriate error message. [A better version, not asked here, would also check if both of the input variables are honest numbers: no alphabetical characters or special symbols].

*In case it helps, below is a sample perl script.*

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```
#!/usr/bin/perl
push(@INC,"/home/httpd/cgi-bin");
require 5.003;
require "cgi-lib.pl";
#----- Sample Perl Script -----
# Input data:  x,  y.      Output: x + y
#----- Main Program -----
&ReadParse;
print &PrintHeader;
$z = $in{x} + $in{y};

print <<"end";
<html><head><title>Math 210, Perl Example 1</title></head>
<body bgcolor=white>
<center><h2> Output for Example 1</h2>

<i>Your input</i>:  <b> x = $in{x}, y = $in{y}</b>
<p>
<i>Answer</i>:  <b>x + y = $z</b>
</center></body></html>
end
```

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