Signature | Printed Name

Math 210 — December 7, 2006

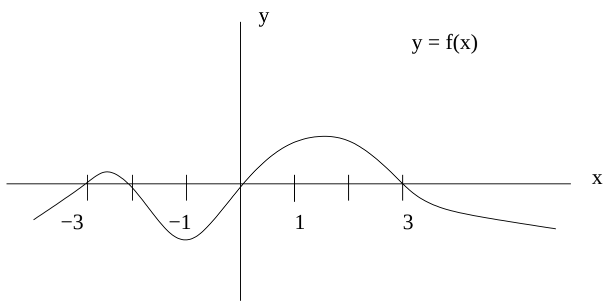
Exam 2

Jerry L. Kazdan — 12:00 — 1:20

Directions: Part A (short answer) has 2 problems (5 points each) while Part B has 7 problems (10 points each). To receive full credit your solution should be clear and correct. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one 3 ×5 with notes on both sides. Please box your answers.

Part A: Short Answer, 10 points (5 points each)

A-1. Suppose $x(t)$ evolves according to the differential equation $dx/dt = f(x)$, where $f(x)$ is the function $f(x)$ graphed below. Describe what happens to $x(t)$ as t gets very large. if $x(0) = 1$.



A-2. Compute an integer c, where $0 \le c < 23$ so that $48 \equiv c \pmod{23}$.

Part B: 70 points (10 points each)

B-1. You and a friend agree to meet for lunch between 12:00 and 1:00 every day. Suppose you both arrive between 12:00 and 1:00, but at times chosen at random.

a) What is the probability distribution function for the amount of time the first to arrive must wait for the other?

b) What is the probability density?

c) What is the expected waiting time?

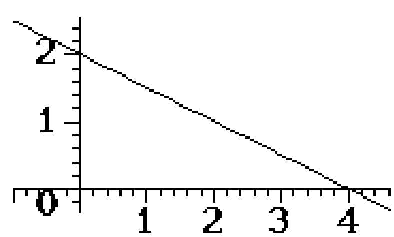
d) What is the standard deviation of the waiting time?

B-2. (see the graph on the right)

a). If the horizontal axis is x and the vertical axis is y, find the equation for y as a function of x?

b). If the horizontal axis is x and the vertical axis is $\log y$, find the equation for y as a function of x.

c). If the horizontal axis is $\log x$ and the vertical axis is $\log y$, find y as a function of x.



B-3. Find a map of the form $F(X) = V + AX$, where A is a 2×2 matrix and V a vector (so F maps the two dimensional plane to itself) that describes a rotation counterclockwise by 90 degrees followed by a reflection across the vertical axis.

B-4. A multinational company has branches in the US., Japan, and Europe. In 1990, it had assets of \$5 million: \$3 million are in the U.S. and \$2 million in Europe. Each year 1/2 the U.S. money stays home, 1/4 goes to both Japan and Europe. For Japan and Europe, 1/2 stays home and 1/2 is sent to the U.S.

a). Find the transition matrix of this Markov chain.

b). Find the limiting distribution of the \$5 million as the world ends.

B-5. Say a function $p(t)$ satisfies $dp/dt = (p-1)(p-3)$. First graph dp/dt in terms of p and use the result to sketch the graphs in the tp plane of $p(t)$ for $t > 0$ under each of the following initial conditions:

$$p(0) = -1, \qquad p(0) = 2, \qquad \text{and} \qquad p(0) = 4.$$

B-6. Let A be a 2×2 matrix with distinct eigenvalues λ_1, λ_2 and corresponding eigenvectors V_1, V_2.

a) If $X = aV_1 + bV_2$, compute AX, A^2X, and $A^{35}X$ in terms of λ_1, λ_2, V_1, V_2, a, and b.

b) If $\lambda_1 = 1$ and $|\lambda_2| < 1$, compute $\lim_{k\to\infty} A^kX$. Explain your reasoning.

B-7. For a crude RSA encryption of a message, you use $n = pq$, where $p = 7$ and $q = 13$.

a). Find a *public exponent* e and a *private exponent* d.

b). Say the entire message Alice want to send you is the number 10. What is Alice's encryption of this message?