Signature   Printed Name

Math 210
Dec. 4, 2008

Exam 2

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9:00 — 10:20

Directions: This exam has 2 parts. Part A has 4 shorter problems (*5 points each*) while Part B has 7 standard problems (*12 points each*). To receive full credit your solution must be clear and correct. You have 2 hours. Closed book, no calculators – but you may use one 3"x5"card with notes. Please box your answers.

Part A, Short Answer (5 points each)

Score	
A-1	
A-2	
A-3	
A-4	
B-1	
B-2	
B-3	
B-4	
B-5	
B-6	
B-7	
Total	

A-1. If you roll a die 7 times, what is the probability of getting at least one of them showing a 5?

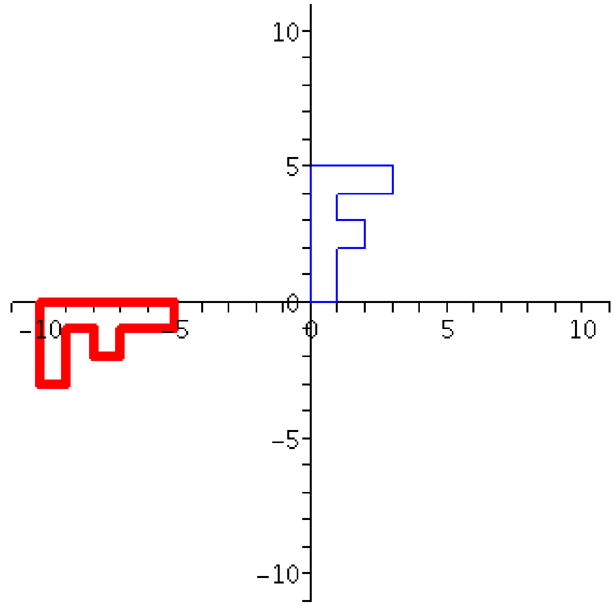
A-2. Write 19 (base 10) as a number base 2.

A-3. If 3^{17} is divided by 5, compute the remainder. Justify your assertion.

A-4. You are given an invertible matrix A. Say V is an eigenvector with eigenvalue 3, so $AV = 3V$. Compute $A^2V + A^{-2}V$.

Part B, Traditional Problems (12 points each)

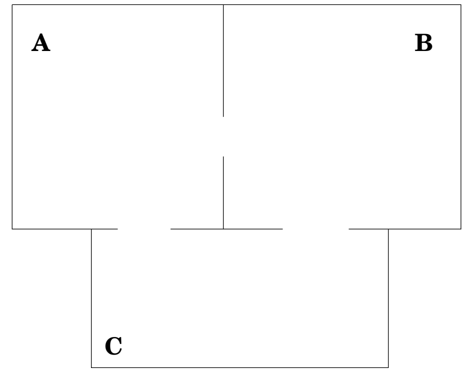
B-1. For the following figure find a matrix A and vector V that gives the indicated transformation $TX = V + AX$. [The new **F** is bold.]



B-2. A house has rooms A, B, and C. Every hour the doors open and you may move to another room — or stay where you are. It was found that:

- A person in room A has a 20% likelihood of moving to room B and a 10% likehood of moving to room C.
- A person in room B has a 20% likelihood of moving to room A and a 20% likehood of moving to room C.
- A person in room C has a 20% likelihood of moving to room A and would never move to room B.

Compute the long-term "popularity" of each room.



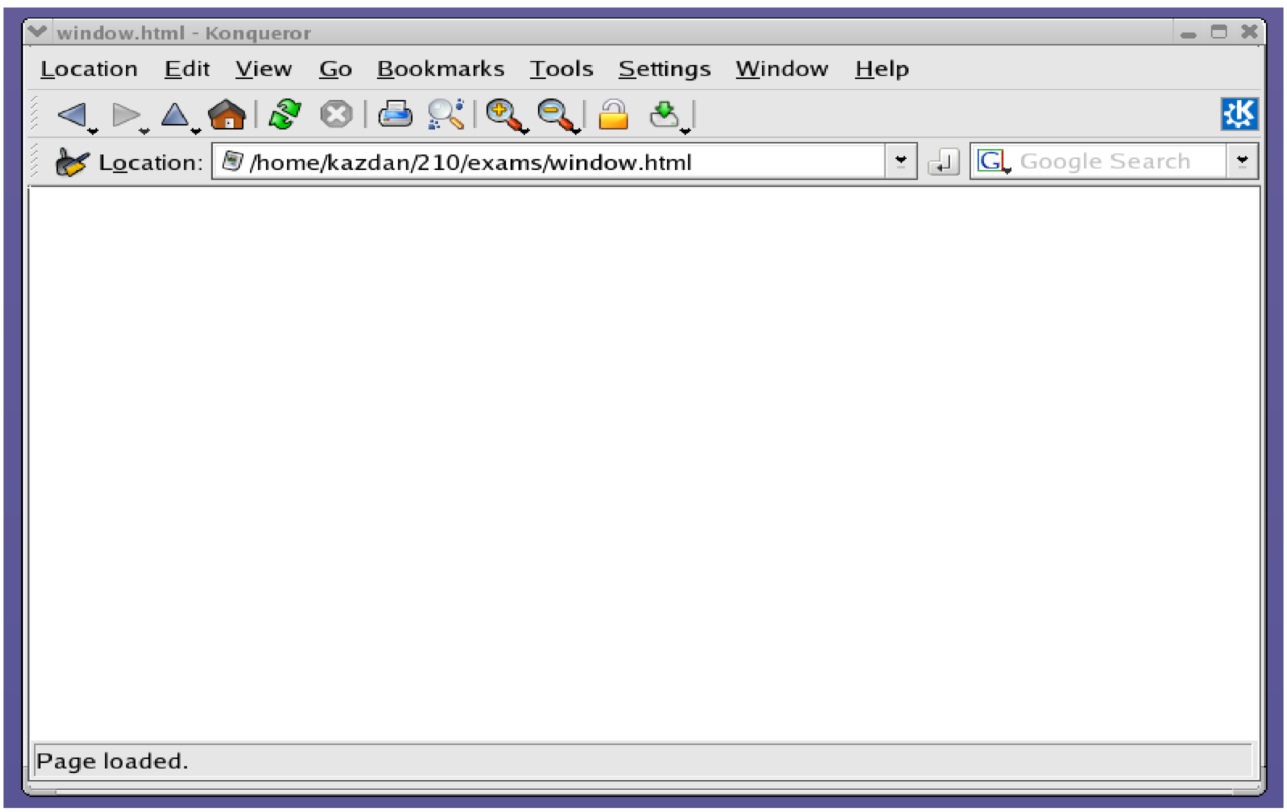
B-3. The following describes a web page. How will it appear? (fill-in the blank page below).

```
<!DOCTYPE HTML PUBLIC "-//W3C//DTD HTML 4.01 Transitional//EN">
<HTML><HEAD>
<meta HTTP-EQUIV="Content-Type" CONTENT="text/html; charset=ISO-8859-1">
<TITLE>Math210 Exam 2</TITLE>
</HEAD>

<BODY BGCOLOR="yellow">
<center><H2>Math 210, Fall 2008 Exam 2</H2></center>

This is our <B>second</B> exam.

      I hope you do well on it.
<P>
Best wishes - and cheers for the coming holidays.
</body></html>
```



B-4. a) Show that for a random variable X, such as the grades x_1, x_2,...,x_n on an exam, the variance $V(X)$ satisfies

$$V(X+c) = V(X) \quad \text{and} \quad V(aX) = a^2 V(X).$$

[If it helps, you may use without proof that the expected value $E(X)$ satisfies $E(X+c) = E(X)+c$ and $E(aX) = aE(X)$.]

b) Say the grades x_1, x_2,...,x_n on an exam have expected value (= mean) $\bar{x}$ and standard deviation C. You want to compute *normalized* grades y_1, y_2,...,y_n of the form $y_j = ax_j + b$ so that these normalized grades have mean $\bar{y}$ and standard deviation D.

Find the coefficients a and b in terma of $\bar{x}$, C, $\bar{y}$ and D. [The algebra will be simpler if you use part a).]

B-5. Let A be a 3×3 matrix with eigenvalues λ_1, λ_2, λ_3 and corresponding (independent) eigenvectors V_1, V_2, V_3 which we can therefore use as a basis. Of course $AV_j = \lambda_j V_j$.

a) If $X = aV_1 + bV_2 + cV_3$, compute AX, A^2X, and $A^{35}X$ in terms of λ_1, λ_2, λ_3, V_1, V_2, V_3, a, b and c (only).

b) If $\lambda_1 = 1$, $|\lambda_2| < 1$, and $|\lambda_3| < 1$, compute $\lim_{k\to\infty} A^k X$. Explain your reasoning clearly.

B-6. You and a friend agree to meet for lunch around noon every Wednesday. Suppose you both arrive between 12:00 and 1:00, but at random times.

a) What is the probability distribution function for the amount of time the first to arrive has to wait for the other? [To get you started, if x is your arrival time and y is your friend's arrival time, then the waiting time $w = |x - y|$].

b) What is the probability density?

c) What is the expected waiting time?

B-7. For a simple RSA encryption, you pick $n = pq$, where $p = 3$ and $q = 11$. Say you use the *public exponent* $e = 3$.

a). Find the *private exponent* d.

b). Say the entire message Alice wants to send you is the number 6. What is Alice's encryption of this message?

c). Breifly *describe* the computation you would need to do to decrypt Alice's message. (You are not asked to complete the calculation).