Math 210   **Final Exam**   Jerry L. Kazdan
May 4, 2001   11:00 — 1:00

DIRECTIONS: This exam has 10 problems (*10 points each*). To receive full credit your solution must be clear and correct. You have 2 hours. Closed book, but you may use one sheet of paper with notes as well as a calculator. This Final Exam counts for 40% of your course grade. Please ⬚box your answers.
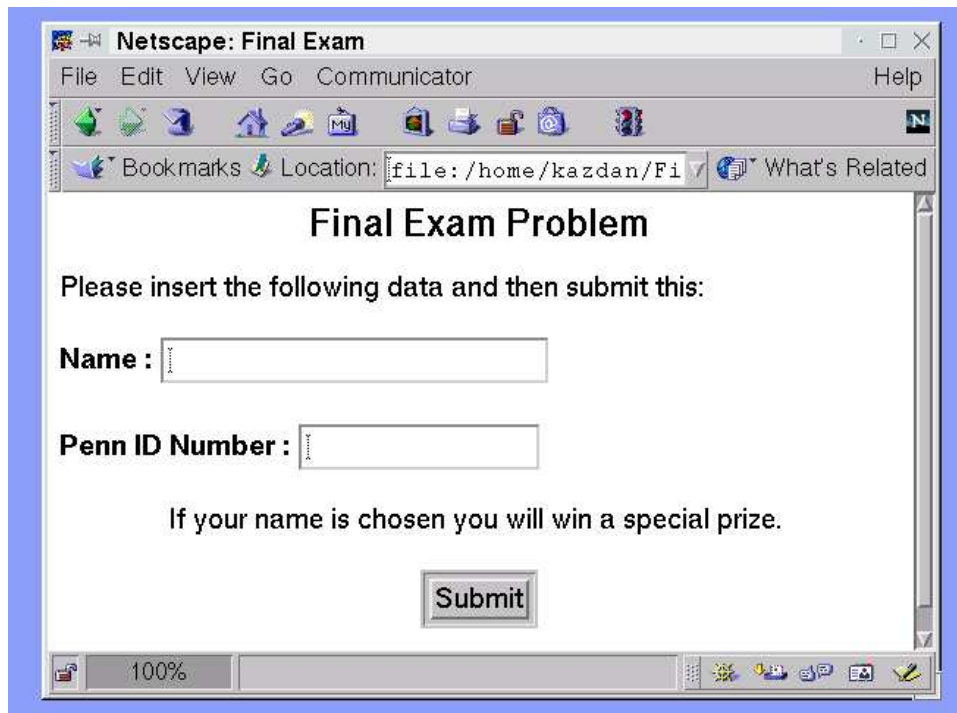
1. (SHORT ANSWER)

    a). Compute 57 (base 10) in base 3.

    b). To whom are you more closely related, your sister's son or your grandfather? Explain.

    c). If you roll a die 3 times, what is the probability of getting at least one of them showing a 6?

    d). Say you have tossed a (fair) coin 99 times and gotten 80 "heads" and 19 "tails". What is the probability that on the next toss it will shown a "head"?

2. This problem concerns **four-sided dice**, with the numbers 1, 2, 3, 4 on the respective faces.

    a). Say you roll a four-sided die once (this is your "event"). To each event you assign the number showing on the die. Compute the expected value and standard deviation.

    b). Say you roll 10 dice once (this is your "event"). To each event you assign the *sum* of the numbers showing on the dice. Compute the expected value and standard deviation.

    c). Say you roll 10 dice once (this is your "event"). To each event you assign the *average* of the numbers showing on the dice. Compute the expected value and standard deviation.

3. Find a map of the form $F(X) = V + AX$, where A is a 2×2 matrix and V a vector that describe each of the following mappings of the plane [the light F is mapped to the dark F].

a).   b).
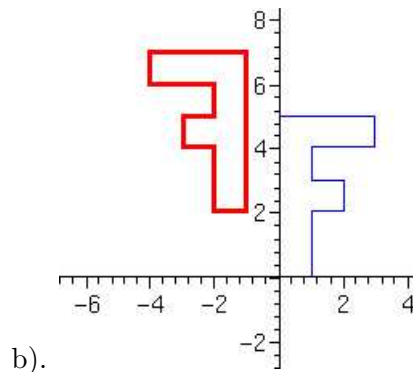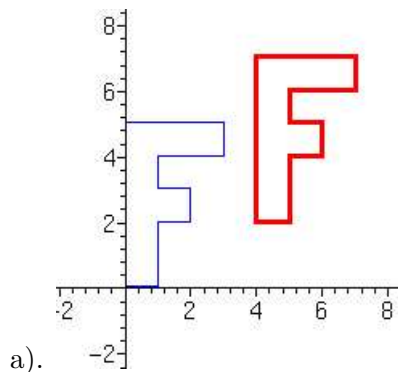
4. This problem and the next one should be read together. In this problem please write an html page that produces the Web Form just below. [For reference, on the last page of this exam there is a form you did earlier this semester.]
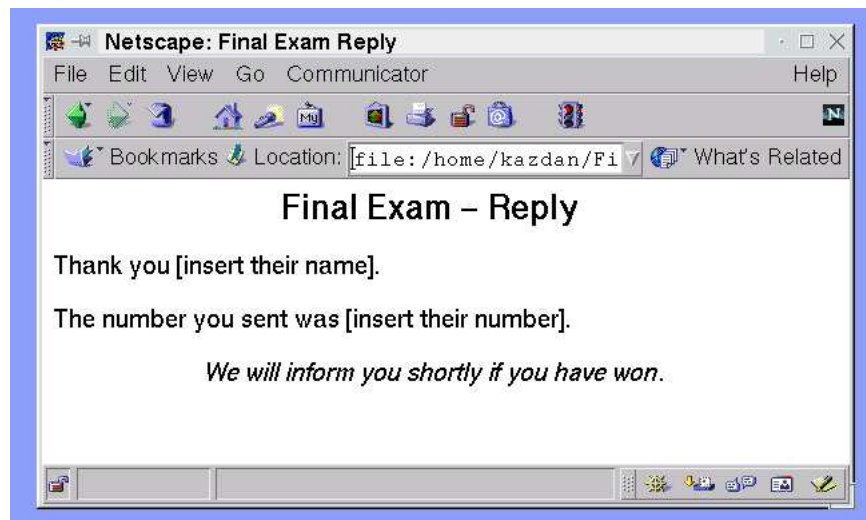
5. This is a continuation of the previous problem. Write a perl script that uses the above data and responds with the following web form: [For reference, on the last page of this exam there is a perl script you did earlier this semester.]

6. Let $T(t)$ be the average temperature of the earth's surfact at time t. Its rate of change primarily depends on the incoming radiation from the sun, $R_{in}(T)$, and the outgoing radiation from ther earth, $R_{out}(T)$:

$$\frac{dT}{dt} = R_{in} - R_{out}.$$

Say $R_{in}(T)$ and $R_{out}(T)$ are given by the following graph (for your convenience they are plotted on the same axes):
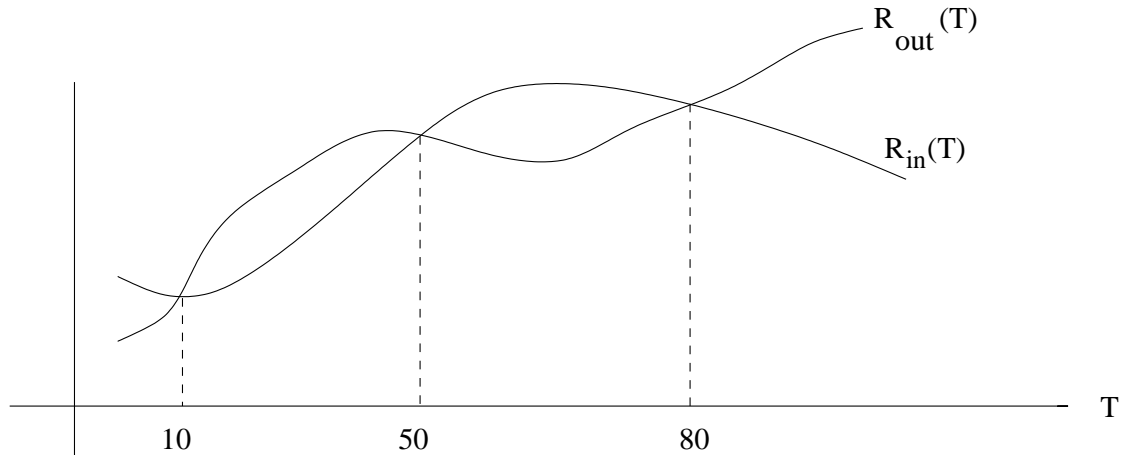
a). For what values of T are there equilibrium points?

b). If $T(0) = 20$, what can you say about the long-term temperature?

c). If $T(0) = 60$, what can you say about the long-term temperature?

d). If $T(0) = 90$, what can you say about the long-term temperature?

7. Consider the following system: $\quad \frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xy - x \\ xy - 2y \end{pmatrix}.$

a). Sketch the null clines and find all the equilibrium points.

b). Determine the stability of all the equilibrium points.

c). If you know that $x(0) = 5$ and $y(0) = 1$, discuss the long-time behavior of $x(t)$ and $y(t)$.

8. Let A be a 2×2 matrix with eigenvalues λ_1, λ_2 and corresponding eigenvectors V_1, V_2.

a) If $X = aV_1 + bV_2$, compute AX, A^2X, and $A^{35}X$ in terms of λ_1, λ_2, V_1, V_2, a, and b (only).

b) If $\lambda_1 = 1$ and $|\lambda_2| < 1$, compute $\lim_{k \to \infty} A^k X$. Explain your reasoning clearly.

9. There are two local branches of the Limousine Rental Company, one at the Airport and one in the City, as well as branches Elsewhere.

   Say every week of the limousines rented from the Airport 25% are returned to the City and 2% to branches located Elsewhere. Similarly of the limousines rented from the City 25% are returned to Airport and 2% to Elsewhere. Finally, say 10% of the limousines rented from Elsewhere are returned to the Airport and 10% to the City.

   If initially there are 35 limousines at the Airport, 35 in the City, and 150 Elsewhere, what is the long-term distribution of the limousines?

10. For a simple RSA encryption, you use $n = pq$, where $p = 5$ and $q = 11$.

    a). Find a *public exponent e* and a *private exponent d*.

    b). Say the entire message Alice want to send you is the number 6. What is Alice's encryption of this message?

Reference: An Old Html and Perl Example

This is an html page that request data for a fill-in form.

```
<html><head><title>Perl Example 1</title></head>
<body bgcolor=white>

<center><h2>Math 210 A simple script using Perl</h2></center>

<form action="/cgi-bin/210/perl_example1.pl">
You specify:
<p>
<center><b>x =</b><input type=text name="x" size=15>
<br><b>y = </b><input type=text name="y" size=15></center>
<p>
This computes  <b>x + y</b>:
<p>
<center><input type=submit value="Submit"></center>
</form>
</body></html>
```

This is the perl script that processes the above.

```
#!/usr/bin/perl
push(@INC,"/home/httpd/cgi-bin");
require 5.003;
require "cgi-lib.pl";
#------------------------ What This Does ---------------------------
# Input data:  x,  y.    Output: x + y
#------------------------- Main Program ---------------------------
&ReadParse;
print &PrintHeader;
$z = $in{x} +  $in{y};

print <<"end";
<html><head><title>Math 210, Perl Example 1 1</title></head>
<body bgcolor=white>
<center><h2> Output for Example 1</h2>

<i>Your input</i>:   <b> x = $in{x}, y = $in{y}</b>
<p>
<i>Answer</i>: <b>x + y = $z</b>
</center></body></html>
end
```