Signature	 Printed Name
Signature	I KINTED NAME

Math 210 March 7, 2002

Mid-Term Exam

Jerry L. Kazdan 1:30 — 2:50

DIRECTIONS: This exam has 7 problems (15 points each). To receive full credit your solution must be clear and correct. No fuzzy reasoning. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one $3'' \times 5''$ card with notes on both sides. Please box your answers.

1. The next two players in a game win 30% and 20% of the time, respectively. What is the likelihood that *neither* of them will win this time? [EQUIVALENT WORDING: It is the ninth inning of a baseball game. There is one out. The batting averages of the next two batters are .300, and .200. Say they face an average pitcher. What is the likelihood that *neither* of them will get a hit this inning?].

Score	
1	
2	
3	
4	
5	
6	
7	
Total	

2. A person takes a test for a relatively rare cancer. This cancer has an incidence of 2% among the general population. Thus, before taking the test, and in the absence of any other evidence, the best estimate of the likelihood of having the cancer is 2 in 100.

Extensive trials have shown that the reliability of this test is 90%. More precisely, it gives a positive result in 10% of the cases where no cancer is present (*false positive*). Moreover, about 5% of the time the test fails to detect the cancer even though it is present (*false negative*).

QUESTION: If Mary tests positive, what is the probability she has the cancer?

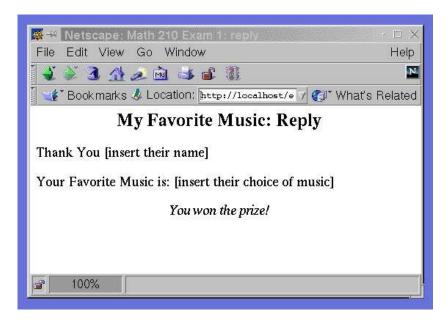
3. This problem and the next one should be read together. In this problem please write an html page that produces the Web Form just below. [For reference, on the last page of this exam there is a form you did earlier this semester.]



Name (print)

3

4. This is a continuation of the previous problem. Write a perl script that uses the above data and responds with the following web form. Note: for each participant your script should pick a random integer between 1 and 5 and print "You won the prize!" if the random number for this person is 5 or otherwise print "Sorry. You didn't win the prize." . [For reference, on the last page of this exam there is a perl script you did earlier this semester.]



5. Say you seek a curve of the *special form* $z = at^4 + b$ to pass through the three data points (-1, 1), (0, 2), (1, 0).

4

a). Write the (over-determined) system of equations you would like to solve ideally for the coefficients a and b.

b). Using the method of least squares write the normal equations for the coefficients a, b.

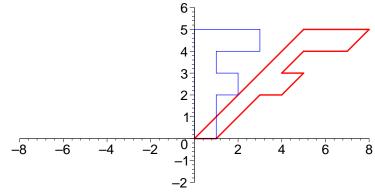
c). Explicitly find the coefficients a, b.

6. There are two local branches of Neff Car Rental Company, one at the Airport and one in the City, as well as branches Elsewhere.

Say every week of the cars rented from the Airport 25% are returned to the City and 5% to branches located Elsewhere. Similarly of the cars rented from the City 25% are returned to Airport and 5% to Elsewhere. Finally, say 10% of the cars rented from Elsewhere are returned to the Airport and 10% to the City.

In the long-term, what percentages of the cars are at the Airport? The City? Elsewhere?

7a). For the following figure find a matrix T that gives the indicated linear transformation.



b). In the above figure, draw the letter **F** that one gets using the *inverse* transformation, T^{-1} . Also compute T^{-1} .

Reference: An Old html and perl example

This is an html page that requests data for a fill-in form.

```
<html><head><title>Perl Example 1</title></head>
<body bgcolor=white>

<center><h2>Math 210 A simple script using Perl</h2></center>
<form action="/cgi-bin/210/perl_example1.pl">
You specify:

<center><b>x = </b><input type=text name="x" size=15>
<br><br><b>y = </b><input type=text name="y" size=15></center>

This computes <b>x + y</b>:

<center><input type=submit value="Submit"></center>
</form>
</body></html>
```

This is the perl script that processes the above.

```
#!/usr/bin/perl
push(@INC,"/home/httpd/cgi-bin");
require 5.003;
require "cgi-lib.pl";
#----- What This Does -----
# Input data: x, y. Output: x + y
#----- Main Program -----
&ReadParse;
print &PrintHeader;
z = \sin\{x\} + \sin\{y\};
print <<"end";</pre>
<html><head><title>Math 210, Perl Example 1 1</title></head>
<body bgcolor=white>
<center><h2> Output for Example 1</h2>
<i>Your input</i>: <b> x = \sin\{x\}, y = \sin\{y\}</b>
>
\langle i\rangle Answer\langle i\rangle: \langle b\rangle x + y = $z\langle b\rangle
</center></body></html>
end
```