Math 210   **Final Exam**   Jerry L. Kazdan

May 1, 2002   11:00 — 1:00

DIRECTIONS: This exam has 9 problems (*10 points each*). To receive full credit your solution must be clear and correct. You have 2 hours. Closed book, no calculators – but you may use one 3"x5"card with notes. Please ⌐box⌐ your answers.

Score	
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

1. (SHORT ANSWER)

a). If 3^{17} is divided by 5, compute the remainder. Justify your assertion.

b). To whom are you more closely related, your brother's son or your mother's sister? Explain.

c). If you roll a die 3 times, what is the probability of getting at least one of them showing a 2?

d). Say you have tossed a (fair) coin 99 times and gotten 83 "heads" and 16 "tails". What is the probability that on the next toss it will shown a "head"?

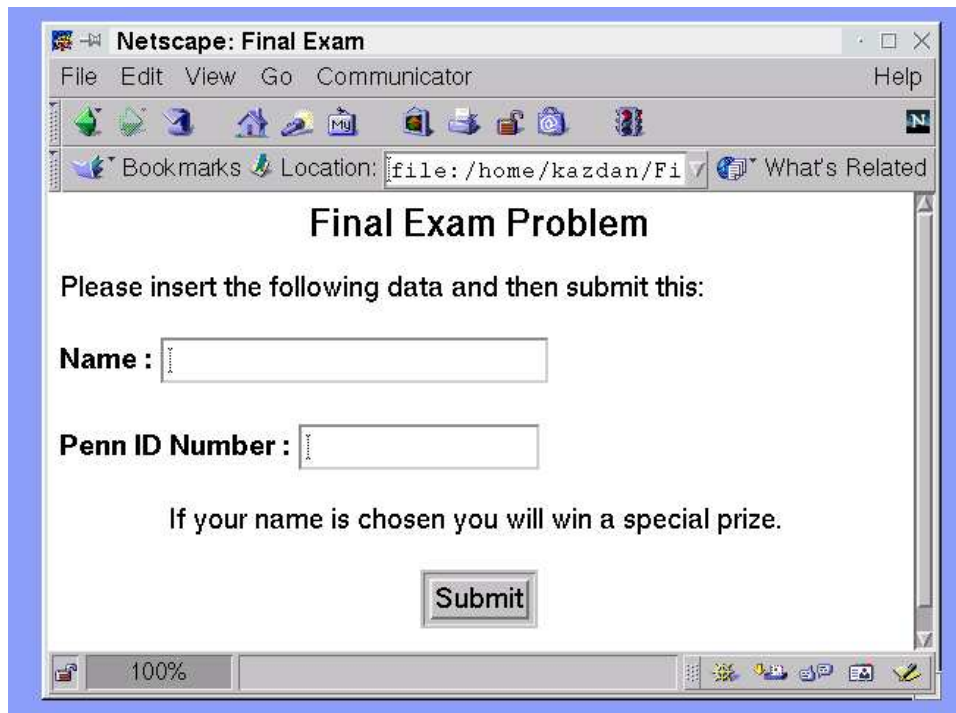2. Discuss which airline has the better "On Time" record?

Destination	Northern Airline			Travel West		
	# arrivals	# on time	% on time	# arrivals	# on time	% on time
Phoenix	200	190	95%	5800	5220	90%
Seattle	2000	1700	85%	400	300	75%
Total	2200	1890	85.9%	6200	5520	89%

[We are seeking a *quantitative* response, not an intuitive one. Since you don't have calculators, approximate calculations will be adequate.]

3. A 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ defines a linear map of the plane to itself, $AX = Y$, where $X = (x_1, x_2)$ and $Y = (y_1, y_2)$. Assume that A is invertible so we know that if two lines are parallel in the $x_1\, x_2$ plane, their images are also parallel. It follows that the image of the unit square Q with vertices at $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ is a parallelogram P with one vertex at the origin.

Show that $\quad \text{Area}\,(P) = \text{Area}\,(Q)\det(A) \quad$ [of course $\text{Area}\,(Q) = 1$].

4. This problem and the next one should be read together. In this problem please write an html page that produces the Web Form just below. [For reference, on the last page of this exam there is a form you did earlier this semester.]

5. This is a continuation of the previous problem. Write a perl script that uses the above data and responds with the following web form: [For reference, on the last page of this exam there is a perl script you did earlier this semester.]
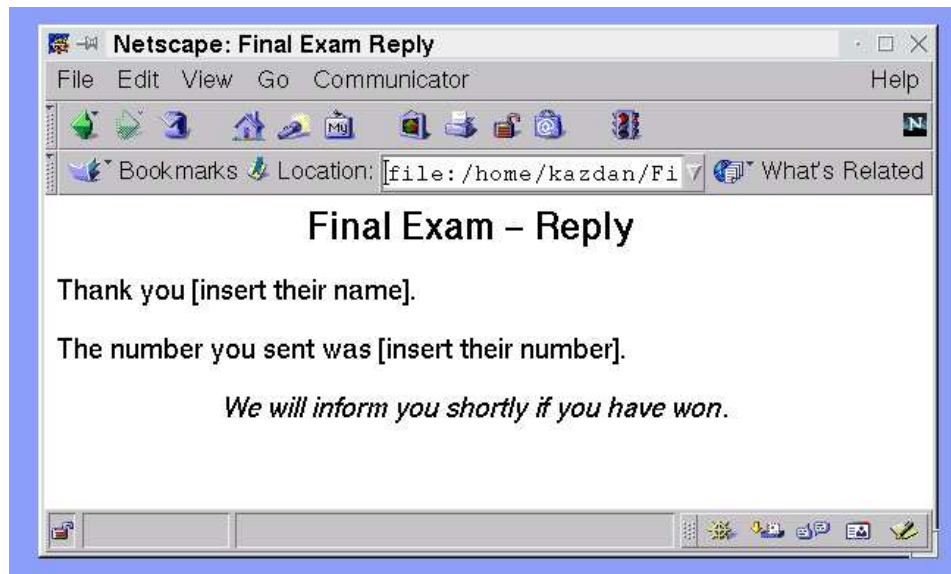
6. A house has rooms A, B, and C. Every hour the doors open and you may move to another room — or stay where you are. It was found that:

   • A person in room A has a 20% likelihood of moving to room B and a 10% likehood of moving to room C.

   • A person in room B has a 20% likelihood of moving to room A and a 20% likehood of moving to room C.

   • A person in room C has a 20% likelihood of moving to room A and would never move to room B.

   Compute the long-term "popularity" of each room.
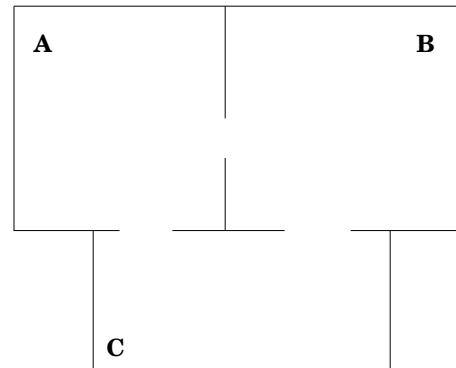
7. Let $T(t)$ be the average temperature of the earth's surfact at time t. Its rate of change primarily depends on the incoming radiation from the sun, $R_{in}(T)$, and the outgoing radiation from ther earth, $R_{out}(T)$:

$$\frac{dT}{dt} = R_{in} - R_{out}.$$

Say $R_{in}(T)$ and $R_{out}(T)$ are given by the following graph (for your convenience they are plotted on the same axes):
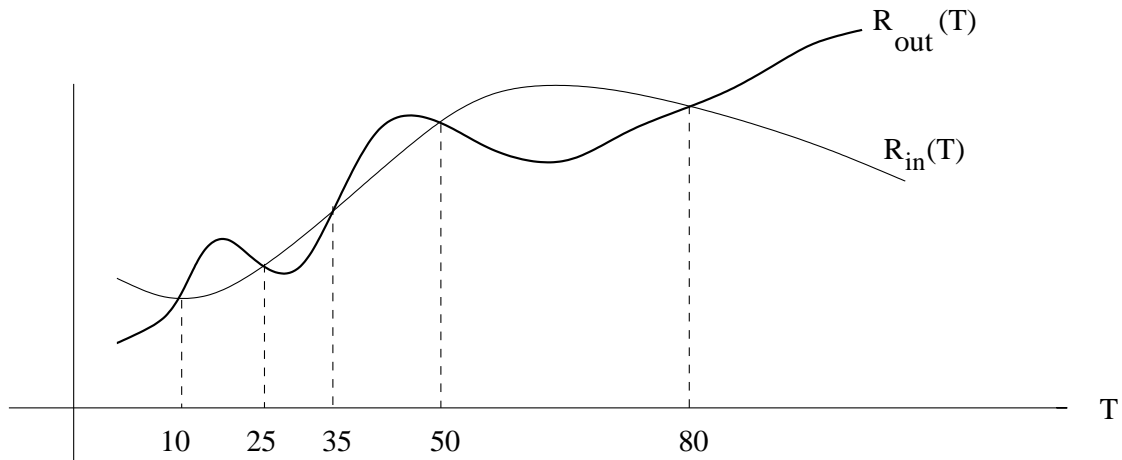
a). For what values of T are there equilibrium points — and which are *stable*?

b). If $T(0) = 20$, what can you say about the long-term temperature?

c). If $T(0) = 60$, what can you say about the long-term temperature?

8. You and a friend agree to meet to play tennis between 11:00 and 12:00 every Wednesday afternoon. Suppose you both arrive between 11:00 and 12:00, but at times chosen at random.

a). What is the probability distribution function for the amount of time the first to arrive has to wait for the other?

b). What is the probability density?

c). What is the expected waiting time?

d). What is the standard deviation of the waiting time?

9. For a simple RSA encryption, you pick $n = pq$, where $p = 5$ and $q = 7$.

    a). Find a *public exponent e* and a *private exponent d*.

    b). Say the entire message Alice wants to send you is the number 8. What is Alice's encryption of this message?

    c). Breifly describe the computation you would need to do to decrypt Alice's message.

Reference: An Old Html and Perl Example

This is an html page that request data for a fill-in form.

```
<html><head><title>Perl Example 1</title></head>
<body bgcolor=white>

<center><h2>Math 210 A simple script using Perl</h2></center>

<form action="/cgi-bin/210/perl_example1.pl">
You specify:
<p>
<center><b>x =</b><input type=text name="x" size=15>
<br><b>y = </b><input type=text name="y" size=15></center>
<p>
This computes  <b>x + y</b>:
<p>
<center><input type=submit value="Submit"></center>
</form>
</body></html>
```

This is the perl script that processes the above.

```
#!/usr/bin/perl
push(@INC,"/home/httpd/cgi-bin");
require 5.003;
require "cgi-lib.pl";
#------------------------ What This Does ----------------------------
# Input data:  x,  y.    Output: x + y
#------------------------- Main Program -----------------------------
&ReadParse;
print &PrintHeader;
$z = $in{x} +  $in{y};

print <<"end";
<html><head><title>Math 210, Perl Example 1 1</title></head>
<body bgcolor=white>
<center><h2> Output for Example 1</h2>

<i>Your input</i>:  <b> x = $in{x}, y = $in{y}</b>
<p>
<i>Answer</i>: <b>x + y = $z</b>
</center></body></html>
end
```