CSE 313
Final Examination
May 7, 2004

Question 1: {20 pts}

Derive an expression for the scalar, γ, as a function of the vectors, $u, v \in R^n$, that will balance the following equation. (Make sure to show your work)

$$(I - uv^T)^{-1} = (I - \gamma(uv^T))$$

Use this result to invert the following matrix:

$$\begin{pmatrix} 2 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 2 & -2 & 1 & -2 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

Question 2: {15 pts}

Determine whether the solutions to the following differential equation are stable or unstable: $\ddot{x} = -5\dot{x} - 4x$

Question 3: {15 pts}

If A is a square matrix prove that the absolute value of the determinant of A, $|\det(A)|$, is equal to the product of its singular values.

Question 4: {10 pts}

If $y \in Range(A)$ and $z \in Null(A^T)$ for some matrix, $A \in R^{m\times n}$, show that $y^T z = 0$.

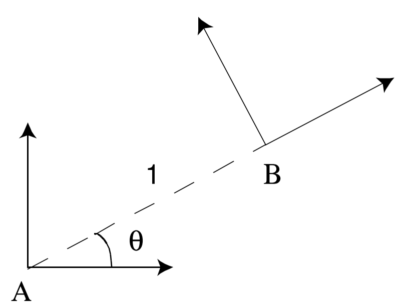
Question 5: {15 pts}

If $\{v_1 \cdots v_n\}$ are orthonormal vectors, $v_i \cdot v_j = \begin{cases} 1 \; if \; i = j \\ 0 \; if \; i \neq j \end{cases}$, and $x = \sum_i \alpha_i v_i$ show that $\|x\|^2 = \sum_i \alpha_i^2$.

Based on this result, explain why dropping small Fourier coefficients is an effective strategy for compressing audio signals.

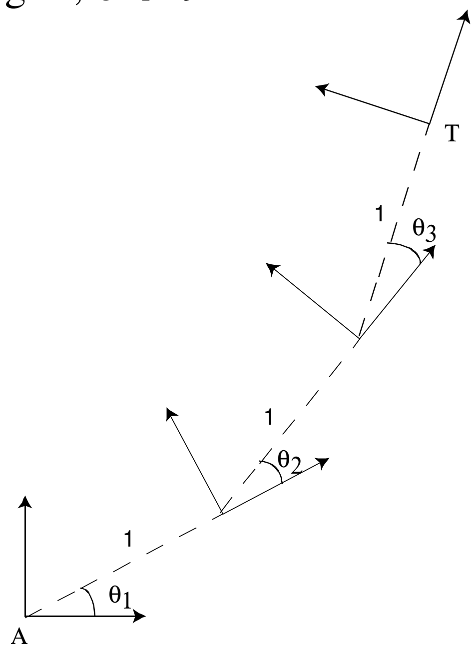
Question 6: {15 pts}

Consider the figure shown below, let x_A, y_A denote the coordinates of a point with respect to frame A and x_B, y_B denote the coordinates of the same point with respect to frame B. These coordinate values can be related by the following equation. $\begin{pmatrix} x_A \\ y_A \\ 1 \end{pmatrix} = g_{AB} \begin{pmatrix} x_B \\ y_B \\ 1 \end{pmatrix}$



Give an expression for the 3 by 3 matrix g_{AB}.

Let g_{AT} denote the 3 by 3 matrix that relates coordinate frames A and T in the figure below. Using your previous result, express g_{AT} as the product of a number of simpler coordinate transformations depending on the angles, $\theta_1, \theta_2, \theta_3$.



Question 7: {15 pts}

If A is a skew-symmetric matrix, $A^T = -A$, show that $x^T Ax = 0$ for all $x \in R^n$

Question 8: {15 pts}

If $R \in R^{n \times n}$ is an orthonormal matrix, $R^T \times R = I$, show that:

1. All of the eigenvalues of R lie on the unit circle in the complex plane, that is $\|\lambda\|^2 = \lambda \times \lambda^* = 1$.
2. All eigenvectors of R are also eigenvectors of R^T .
3. If $v_1, v_2 \in R^n$ are eigenvectors of R corresponding to distinct eigenvalues then they must be orthogonal,