Math 425, Spring 2015 **Jerry L. Kazdan**

Problem Set 2

DUE: Thurs. Jan. 29 in class. [Late papers will be accepted until 1:00 PM Friday.]

This week. Please read all of Chapter 2 in the Strauss text.

MORE RUST REMOVER *These problems are not to be handed-in*

RR 1-12 Problems 1-12 on pages 5-6 of Strauss. You should be able to do most of these instantly.

RR-13 Say the temperature in the plane $\mathbb{R}^2$ at time t is given by

$$u(x, y, t) = 1 - x + 3y^2 + xyt.$$

If you are at the point $(1, 2)$ at time $t = 5$, in what direction in $\mathbb{R}^2$ should you move so the the temperature increases most? Decreases most?

RR-14 Let $\mathbf{V} = (x+2y^2)\mathbf{i}+(2+3y\mathbf{j}$ be a vector field in the plane and let D be the unit disk whose boundary, B is of course the unit circle. Use the divergence theorem to compute

$$\oint_B \mathbf{V} \cdot \mathbf{N}\, ds,$$

where $\mathbf{N}$ is the unit outer normal vector field on B.

RR-15 Strauss, p. 19 #9

The following problems are all from the Strauss text. These are to be handed-in.

1. p. 9 #2

2. p. 10 #8 There are two good methods. Both are valuable to know.
   a) Make a change of the *independent variables* $s = \alpha x + \beta y$, $t = \gamma s + \delta y$, just in in HW-1 #11 to reduce this to a problem like HW-1 #9.
   b) Make a change of the *dependent variable*, say $u(x, y) = \varphi(x)v(x, t)$ choosing φ so that the equation for v has the simpler form $av_x + bv_y = 0$.

3. p. 10 #10 [See the suggestion for the previous problem]

4. p. 19 #6 [For the Laplacian in polar coordinates see p. 157 Eq. (5). Here you are seeking a solution that does not depend on the angle θ.]

5. p. 19 #8

6. p. 20 #10

7. p. 27 #1

8. p. 27 #4 [See also the next problem].

9. Let $D \in \mathbb{R}^3$ be a bounded region with smooth boundary ∂D and say $u(x, y, z)$ satisfies $\Delta u = 0$ in D with $\partial u/\partial n = 0$ on the boundary. Show that $u(x, y, z)$ =constant in D.

   [NOTE: This is almost identical to Problem 16 in Homework Set 1.

10. p. 38 #1

11. p. 38 #3

12. p. 38 #7

Bonus Problem

[Please give this directly to Professor Kazdan]

B-1 Strauss P. 41 #6

[Last revised: January 23, 2015]