$\boxed{\text{p. 9 \#2}}$

$3u_y + u_{xy} = 0$

$v = u_y$

$\Rightarrow 3v + v_x = 0$

$\Rightarrow v(x,y) = g(y)e^{-3x}$

$\Rightarrow u_y = g(y)e^{-3x}$

$\Rightarrow u = e^{-3x}\int g(y)\,dy + h(x)$ $\square$

$\boxed{\text{p. 10 \#8}}$

a) $s = \alpha x + \beta y$

$t = \gamma x + \delta y$

$u_x = u_s \frac{\partial s}{\partial x} + u_t \frac{\partial t}{\partial x} = \alpha u_s + \gamma u_t$

$a u_x + b u_y + c u = 0$

becomes $(a\alpha + b\beta) u_s + (a\gamma + b\delta) u_t + cu = 0$

want $a\gamma + b\delta = 0$

$\gamma = b \quad \delta = -a$

$\alpha = a \quad \beta = b$

$\Rightarrow (a^2 + b^2) u_s + cu = 0$

$u_s = \frac{-c}{a^2+b^2} u$

$u = g(t) \exp\left(\left(\frac{-c}{a^2+b^2}\right) s\right)$

$\Rightarrow u(x,y) = g(bx - ay) \exp\left(\frac{-c}{a^2+b^2}(ax+by)\right)$ $\square$

b) $u(x,y) = \varphi(x)\, v(x,y)$

$u_x = \varphi' v + \varphi v_x$

$u_y = \varphi v_y$

$\Rightarrow a\varphi' v + a\varphi v_x + b\varphi v_y + c\varphi v = 0$

want $\quad a\varphi' v = c\varphi v$

$$a\varphi' = c\varphi$$

$$\varphi = Ae^{-\frac{c}{a}x}$$

A can be anything we want!

$$\Rightarrow \cancel{-Aa\frac{c}{a}e^{-\frac{c}{a}x}v} + Aae^{-\frac{c}{a}x}v_x + Abe^{-\frac{c}{a}x}v_y + \cancel{Ace^{-\frac{c}{a}x}v} = 0$$

$$\Rightarrow Aae^{-\frac{c}{a}x}v_x + Abe^{-\frac{c}{a}x}v_y = 0$$

$$av_x + bv_y = 0$$

$$\Rightarrow v(x,y) = g(bx - ay)$$

$$\Rightarrow u(x,y) = g(bx-ay)e^{-\frac{c}{a}x}$$

$\square$

$\boxed{\text{p. 10 \#10}}$

Same as (b)

$$u_x + u_y + u = e^{x+2y}$$

$$\varphi' v + \varphi v_x + \varphi v_y + \varphi v = e^{x+2y}$$

want $\varphi' = -\varphi$

$\varphi = Ae^{-x}$ Free to choose A!

$$\Rightarrow -Ae^{-x}v + Ae^{-x}v_x + Ae^{-x}v_y + Ae^{-x}v = e^{x+2y}$$

$$v_x + v_y = \frac{1}{A}e^{2x+2y}$$

Look for particular solution of "similar form"

$$v = e^{2x+2y}$$

$$v_x = 2e^{2x+2y} \quad v_y = 2e^{2x+2y}$$

$$v_x + v_y = 4e^{2x+2y}$$ so if we set $A = \frac{1}{4}$, we're good!

$$\varphi = \frac{1}{4}e^{-x} \quad v(x,y) = e^{2x+2y}$$

$$\Rightarrow u = \varphi v = \frac{1}{4}e^{x+2y}$$ is particular solution.

homogeneous ~~General~~ solution is

$$v_x + v_y = 0 \quad v = f(x-y)$$

$$u = \varphi v = \frac{1}{4}e^{-x}f(x-y)$$

General solution is

$$u(x,y) = \frac{1}{4}e^{x+2y} + \frac{1}{4}e^{-x}f(x-y)$$

Initial Condition: $0 = u(x,0) = \frac{1}{4}e^{x} + \frac{1}{4}e^{-x}f(x) \Rightarrow f(x) = \dfrac{-\frac{1}{4}e^{x}}{\frac{1}{4}e^{-x}} = -e^{2x}$

$\Rightarrow f(x-y) = -e^{2x-2y}$

$$\Rightarrow u(x,y) = \frac{1}{4}e^{x+2y} - \frac{1}{4}e^{x-2y}$$

$\square$

$\boxed{\text{p. 19 \#6}}$

$$u_t = k\Delta u$$
$$= k\left[u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right] \quad \text{by p. 157 (5)}$$

By assumption, $u_\theta = 0$

$$\Rightarrow \quad u_t = k\left[u_{rr} + \frac{1}{r}u_r\right] \qquad \square$$

$\boxed{\text{p. 19 \# 8}}$

$$\int |u|^2 dx = 1 \quad \text{at } t = 0$$

$$\int \frac{\partial}{\partial t} |u|^2 dx = \int \frac{\partial}{\partial t} u\bar{u}\, dx = \int u_t \bar{u} + \bar{u}_t u \, dx$$

u satisfies Schrödinger's equation

$$-ihu_t = \frac{h^2}{2m} \Delta u + \frac{e^2}{r} u$$

$$\Rightarrow u_t = \frac{ih}{2m} \Delta u + \frac{ie^2}{hr} u$$

$$\Rightarrow \bar{u}_t = \frac{-ih}{2m} \overline{\Delta u} - \frac{ie^2}{hr} \bar{u}$$

$$\int \left(\frac{ih}{2m} \Delta u + \frac{ie^2}{hr} u\right)\bar{u} - \left(\frac{ih}{2m} \overline{\Delta u} + \frac{ie^2}{rh} \bar{u}\right) u \, dx$$

$$= \frac{ih}{2m} \int \Delta u \bar{u} - \overline{\Delta u} u \, dx$$

$$= \frac{ih}{2m} \int \nabla \cdot [\nabla u \bar{u} - \nabla \bar{u} u] \, dx$$

$$= \frac{ih}{2m} \int_{\substack{\partial B(0,R) \\ \text{for } R \gg 1}} [\nabla u \bar{u} - \nabla \bar{u} u] \cdot \vec{n} \, dS \quad \text{by Div. Thm}$$

$\to 0$ as $R \to \infty$ as $u, \nabla u \to 0$ "fast enough".

$$\therefore \frac{d}{dt} \int |u|^2 dx = 0 \Rightarrow \int |u|^2 dx = 1 \ \forall t$$

$\square$

$\boxed{\text{p. 20 \#10}}$

$$\int_{B(0,R)} \nabla \cdot f(x)\, dx = \int_{\partial B(0,R)} F \cdot \vec{n}\, ds$$

$R \gg 1$

$$\left| \int_{\partial B(0,R)} f \cdot \vec{n}\, ds \right| \leq \int_{\partial B(0,R)} |f \cdot \vec{n}|\, ds$$

$$\leq \frac{1}{R^3+1} \text{SArea}(\partial B)$$

$$= \frac{1}{R^3+1} 4\pi R^2$$

as $R \to \infty \to 0$

Therefore, $\int_{\mathbb{R}^3} \nabla \cdot f\, dx = 0$

$\boxed{\text{p. 27 \#1}}$

$u'' + u = 0$

$u(x) = c_1 \cos(x) + c_2 \sin(x)$

$u(0) = 0 \Rightarrow c_1 + 0 = 0 \Rightarrow c_1 = 0$

$u(x) = c_2 \sin(x)$

$u(L) = 0$

$0 = c_2 \sin(L)$

Not necessarily unique depending on value of L.
(ex: $L = 0, \pi, 2\pi, \ldots$)

$\boxed{\text{p. 27 \#4}}$

$\Delta u = f(x,y,z)$ in D

$\frac{\partial u}{\partial n} = 0$ on ∂D

(a) Add a constant doesn't change solution.

(b) $\iiint_D F \, dxdydz = \iiint \Delta u \, dxdydz$

$= \iiint \nabla \cdot \nabla u \, dxdydz$

$= \iint_{\partial D} \nabla u \cdot \vec{n} \, dS = \iint_{\partial D} \frac{\partial u}{\partial n} \, dS = 0.$

9)

$$\iiint_D u\Delta u \, dx\,dy = \int_D \nabla\cdot(u\nabla u) - |\nabla u|^2 d\vec{x} = 0$$

as $\Delta u = 0$ in D.

By Div. Thm,

$$\int_D \nabla\cdot(u\nabla u)\, d\vec{x} = \int_{\partial D} u\nabla u\cdot\vec{n}\, dS$$

$$= \int_{\partial D} u\frac{\partial u}{\partial n}\, dS = 0 \quad \text{by assumption.}$$

Therefor $-\int_D |\nabla u|^2 d\vec{x} = 0 \Rightarrow \nabla u = 0 \Rightarrow u$ is constant. □

$\boxed{\text{p.38 \#1}}$

$\varphi(x) = e^x \quad \psi(x) = \sin x$

$$u(x,t) = \frac{1}{2}\left[e^{x+ct} + e^{x-ct}\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} \sin(x)\, dx$$

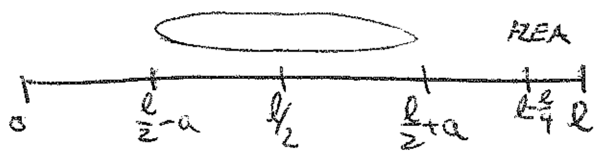
$$= \frac{1}{2}\left[e^{x+ct} + e^{x-ct}\right] + \frac{1}{2c}\left[\cos(x-ct) - \cos(x+ct)\right]$$

□

p 38 #3

By example (2), $c = \sqrt{\frac{T}{\rho}}$

As head diameter $= 2a$, head hits all the way to $\frac{\ell}{2} + a$



distance is $\ell - \frac{\ell}{4} - \frac{\ell}{2} - a = \frac{\ell}{4} - a$.

$$\text{time} = \frac{\frac{\ell}{4} - a}{\sqrt{\frac{T}{\rho}}} = \sqrt{\frac{\rho}{T}}\left(\frac{\ell}{4} - a\right) \qquad \square$$

p. 38 #7

$$u(x,t) = \frac{1}{2}\left[\varphi(x+ct) + \varphi(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(s)\,ds$$

$$\left.\begin{aligned} \varphi(x) &= -\varphi(-x) \\ \psi(x) &= -\psi(-x) \end{aligned}\right\} \text{ as odd.}$$

$$u(-x,t) = \frac{1}{2}\left[\varphi(-x+ct) + \varphi(-x-ct)\right] + \frac{1}{2c}\int_{-x-ct}^{-x+ct} \psi(s)\,ds$$

$$= -\frac{1}{2}\left[\varphi(x-ct) + \varphi(x+ct)\right] + (\quad)$$

$$= [\quad] +$$

let $u = -s$
$du = -ds$
$-x+ct \rightarrow x-ct$
$-x-ct \rightarrow x+ct$

$$= -\frac{1}{2}\left[\varphi(x-ct) + \varphi(x+ct)\right] + \frac{1}{2c}\int_{x+ct}^{x-ct} -\psi(-u)\,du$$

$$= -\frac{1}{2}\left[\varphi(x-ct) + \varphi(x+ct)\right] - \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(u)\,du = -u(x,t) \qquad \square$$