Math 425, Spring 2015 **Jerry L. Kazdan**

Problem Set 9

DUE: Thurs. April 2 in class. [Late papers will be accepted until 1:00 PM Friday.]

This week. Please read the Chapter 9 in the Strauss text.

1. Strauss, p. 196 #3
2. Strauss, p. 196 #5
3. Strauss, p. 196 #6
4. Strauss, p. 197 #11
5. Strauss, p. 197 #13
6. Strauss, p. 233 #1
7. Strauss, p. 233 #2
8. Strauss, p. 233 #6(a)

Bonus Problem

[Please give this directly to Professor Kazdan]

B-1 a) Let A be a positive definite $n \times n$ real matrix (so, by definition, A is assumed to be a symmetric matrix), and $b \in \mathbb{R}^n$. Consider the quadratic polynomial

$$Q(x) := \tfrac{1}{2}\langle x, Ax\rangle - \langle b, x\rangle.$$

Show that Q is bounded below, that is, there is a constant m so that $Q(x) \geq m$ for all $x \in \mathbb{R}^n$.

b) If $x_0 \in \mathbb{R}^n$ minimizes Q, show that $Ax_0 = b$. [Moral: One way to solve $Ax = b$ is to minimize Q.]

c) Let $\Omega \in \mathbb{R}^n$ be a bounded region with smooth boundary and $F(x)$ a bounded continuous function. Also, let $\mathcal{S}$ be the set of smooth functions $u(x)$ on Ω that are zero on the boundary, $u(x) = 0$ for all $x \in \partial\Omega$. Define

$$J(u) := \iint_\Omega [\tfrac{1}{2}|\nabla u|^2 + F(x)u]\, dx.$$

If $u_0(x) \in \mathcal{S}$ minimizes $J(u)$ for all $u \in \mathcal{S}$, show that $\Delta u_0 = F$ in Ω – and of course $u_0 = 0$ on $\partial\Omega$. [Moral: One way to solve $\Delta u = F$ with $u = 0$ on $\partial\Omega$ is to seek a function in $\mathcal{S}$ that mimimizes $J(u)$.]

[Last revised: April 14, 2015]