MATH 240 QUIZ 9 Name: ____________________

Question:

Solve the differential equation:

$$y'' + y' + y = x^2 + x + 1$$

Solution:
First solve for the homogeneous equation:

$$y_c'' + y_c' + y_c = 0$$

its auxilary polynomial $r^2 + r + 1$ has two roots: $r = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$, therefore the general solutions are:

$$y_c = C_1 \mathrm{e}^{-\frac{1}{2}x} \cos \frac{\sqrt{3}}{2} x + C_2 \mathrm{e}^{-\frac{1}{2}x} \sin \frac{\sqrt{3}}{2} x$$

Next we solve for a particular solution y_p, we can use trial solution $y_p = Ax^2 + Bx + C$, since

$$\begin{aligned} y_p'' + y_p' + y_p &= 2A + 2Ax + B + Ax^2 + Bx + C \\ &= Ax^2 + (2A + B)x + (2A + B + C) \end{aligned}$$

compare it with coefficients of $x^2 + x + 1$, we need $A = 1$, $2A + B = 1$, $2A + B + C = 1$. Therefore $A = 1, B = -1, C = 0$, and the particular solution is $y_p = x^2 - x$.
The general solution to the given equation is

$$\begin{aligned} y &= y_c + y_p \\ &= C_1 \mathrm{e}^{-\frac{1}{2}x} \cos \frac{\sqrt{3}}{2} x + C_2 \mathrm{e}^{-\frac{1}{2}x} \sin \frac{\sqrt{3}}{2} x + x^2 - x \end{aligned}$$