MATH 361 — HOMEWORK 12.

due on Thursday, December 10.

Textbook: *"Elementary Classical Analysis"* , second edition
by J. E. Marsden and M. J. Hoffman

Topics:

- **Chapter 6:** Differentiable Mappings
  - 6.8 Taylor's Theorem and Higher Derivatives
  - 6.9 Maxima and Minima
- **Chapter 7:** The Inverse and Implicit Function Theorems and Related Topics
  - 7.1 Inverse Function Theorem

Twelvth Homework Assignment.

Reading:

- Read Sections 6.9 and 7.1. Read the slides (or/and watch the videos).

Exercises: *(In what follows E and F are Banach Spaces).*

Problem 1. Prove that the inverse map $\mathrm{Inv} : \mathrm{GL}(E,F) \to \mathrm{GL}(F,E) \subset \mathrm{L}(F,E)$ is of class C^∞ and that the $n-$th derivative is given by the formula

$$\mathrm{D}^n\mathrm{Inv}(X)(H_1,\dots,H_n) = (-1)^n \sum_{\sigma\in S_n} X^{-1}H_{\sigma(1)}X^{-1}\cdots X^{-1}H_{\sigma(n)}X^{-1},$$

for every $X \in \mathrm{GL}(E,F)$ and every $H_1,\dots,H_n \in \mathrm{L}(E,F)$.

(Use Induction and either the formula for the first derivative or the series for $(X+H)^{-1}$ proven in class.)

Problem 2. Prove that the Taylor polynomial of order n of Inv about $X \in \mathrm{GL}(E,F)$ is given by

$$\mathrm{T}^n_{\mathrm{Inv},X}(H) = X^{-1} - X^{-1}HX^{-1} + X^{-1}HX^{-1}HX^{-1} + \cdots + (-1)^n X^{-1}(HX^{-1})^n.$$

***Problems*:**

- Page 367: $1, 3, 6$
- Page 396: problems: $1, 3$
- Page 438: problems: $3, 5$