MATH 361 — HOMEWORK 3.

due on Friday, September 25.

Textbook: *"Elementary Classical Analysis"* , second edition
by J. E. Marsden and M. J. Hoffman

Topics:

- **Review of Math 360**
- **5. Uniform Convergence**
  - 5.1 Pointwise and Uniform Convergence
  - 5.2 The Weierstrass M Test
  - 5.5 The Space of Continuous Functions
  - 5.6 The Arzela-Ascoli Theorem
  - 5.7 The Contraction Mapping Principle and Its Applications

Third Homework Assignment.

Reading:

- Read section 5.7 of Chapter 5., paying close attention to the examples. Read your notes.

Exercises:

Problem 1. Let $B = \{ f \in C_b(\mathbb{R}, \mathbb{R}) \mid f(x) > 0 \text{ for all } x \in \mathbb{R} \}$. What is the closure of B.?

Problem 2. Let $\mathcal{B} \subset C([0,1], \mathbb{R}^N)$ be closed, bounded, and equicontinuous. Let $I : \mathcal{B} \to \mathbb{R}$ be defined by

$$I(f) = \| \int_0^1 f(x)\, dx \|.$$

Show that there is an $f_0 \in \mathcal{B}$ at which the value of I is maximized.

Does the statement remain true if we replace $\mathbb{R}^N$ with a general Banach space $(E, \| \, \|)$?

Problem 3. Let the functions $f_n : [a,b] \to \mathbb{R}^N$ be uniformly bounded continuous functions (that is $\sup_n \|f_n\|_\infty < +\infty$). Set

$$F_n(x) = \int_a^x f_n(t)\, dt, \qquad a \le x \le b.$$

Prove that F_n has a uniformly convergent subsequence.

Does the statement remain true if we replace $\mathbb{R}^N$ with a general Banach space $(E, \| \, \|)$?

***Problems*:**

- Page 283: problems 8.
- Page 316: problems $11, 14, 23, 33, 50$.