MATH 210, PROBLEM SET 4

DUE IN LECTURE ON THURSDAY, MARCH 22.

1. The B.S. model with both a credibility and a lying benefit

In class we discussed the speaker versus listener game when the listener assigns a credibility benefit, as well as the case in which the speaker assigns a lying benefit. This problem is about the case in which these benefits both exist and are equal to a constant ℓ in the range $0 < \ell < 1$. Here 1 stands for the benefit to the speaker if a scandal is not believed to have happened by the listener.

4. Explain why the payoff matrix to the speaker is

$$A = \begin{pmatrix} -1 & -\ell & 1-2\ell \\ \ell & 0 & -\ell \\ 1+2\ell & \ell & -1 \end{pmatrix}$$

where as usual, the rows stand for the speaker's options of telling the truth, B.S.-ing or lying, and the columns stand for the listener thinking what was said is the truth, thinking it is B.S. and thinking it is a lie.

5. Assume $0 < \ell < 1$ from now on. Is there a dominant strategy?

6. By adding $\delta = 2$ to every entry of A, one gets a matrix A' with positive entries. Write down the linear programming problem involving $s = (s_1, s_2, s_3)$ which is associated to A'.

7. Show that the sum of the first and third columns of A' equals twice the middle column. Use this to show that every vertex $s = (s_1, s_2, s_3)$ of the linear programming problem must have at least one entry equal to 0.

8. Find all vertices of the linear programming problem for which two of s_1, s_2, s_3 equal 0, and calculate $f(s) = s_1 + s_2 + s_3$ for these vertices.

9. Find the remaining vertices, and determine the optimal vertices.

Hints: Show that for the remaining vertices, all entries of sA' are equal to 1, and that setting any two of these entries equal to 1 implies that the third is equal to 1. Thus there are two independent equations, and you get a third by setting one of s_1, s_2 or s_3 equal to 0. To find the solutions, you may find it useful to use Wolfram alpha:

https : //www.wolframalpha.com/examples/math/algebra/

Click on the link under "equation solving" and then the link about solving systems of linear equations. If you label (s_1, s_2, s_3) by (x, y, z), you can solve linear equations in which there is a parameter c.

10. What are all the optimal strategies for the speaker in this game?