MATH 350: HOMEWORK #6

DUE IN JIANRU'S MAILBOX BY DEC. 8, 2014.

1. Applications of continued fractions

1. The number of days it takes the moon to reach the same position in the sky relative to the position of sun is 29.530588853. Suppose one wanted to devise a calendar in which each month had a whole number of days, and at the start of each month the position of the moon was as close to being in same position relative to the sun as at the start of the last month. How would you use the convergents of the continued fraction of 29.530588853 to devise a system of varying the number of days in each month in a minimal way so as to achieve this? What are the results of doing this for more and more convergence? You can use Wolfram alpha to do calculations. The function "continued fraction(r)" gives the continued fraction of a real number r, while "convergence(continued fraction(r))" gives the convergents of this continued fraction.

2. The fast Fourier transform and factoring

Suppose $M \geq 1$ and that $F : \{0, \ldots, M-1\} \to \mathbb{C}$ is a function. We defined $\zeta_M = e^{2\pi\sqrt{-1}/M}$ and the Fourier transform $\hat{F} : \{0, \ldots, M-1\} \to \mathbb{C}$ by

$$\hat{F}(j) = \sum_{i=0}^{M-1} F(i)\zeta_M^{-ij}.$$

2. Suppose $M = 2N$ for some integer N. The fast Fourier transform breaks the computation of the Fourier transform $\hat{f}$ of a function $f : \{0, \ldots, 2N-1\} \to \mathbb{C}$ into the computation of the Fourier transforms $\hat{f}_{even}$ and $\hat{f}_{odd}$ of two functions f_{even} and f_{odd} on $\{0, \ldots, N-1\}$. Now suppose $M = 3N$ for some N. What sorts of formulas would you get for $\hat{f}$ if you started instead with a function $f : \{0, \ldots, 3N-1\}$, and you broke the computation into the computing the Fourier transforms of three functions $f_i : \{0, \ldots, N-1\}$ for $i = 0, 1, 2$ using the integers in $\{0, \ldots, 3N-1\}$ which are congruent to i mod 3?
3. Suppose ℓ and N are positive integers and that ℓ divides N. Suppose $f : \{0, \ldots, N\} \to \mathbb{C}$ is a function, and continue f to a function on all of $\mathbb{Z}$ by making it periodic mod N. Suppose that the resulting f is in fact periodic mod ℓ. Prove that if $\hat{f} : \{0, \ldots, N-1\} \to \mathbb{C}$ is the Fourier transform of f, then $\hat{f}(j) = 0$ unless j is a multiple of N/ℓ. (This is an example of how to read off the true period of a function from its largest Fourier coefficients, which is a key part of factoring with Fourier transforms.)