Math 103
Final Exam Fall 2019

First and Last Name _____(PRINT) Penn ID_____

Professor **(circle one):** Rimmer Villano

This exam has 14 questions. Each question is worth 10 points each for a total of 140 points. Partial credit will be given for the entire exam so be sure to show all work. Circle the correct answer and give supporting work, **a correct answer with little or no supporting work will receive little or no credit.** Use the space provided to show all work. One sheet of scrap paper is provided at the end of the exam, do not rip it off. If you write on these scrap pages or the back of any page please and expect that to be graded, indicate this in some strong way.

You have **120 minutes** to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Once you have completed the exam, sign the academic integrity statement below.
Do **NOT** write in the grid below. It is for grading purposes only.

Problem	Points	Problem	Points
1	/10	**8**	/10
2	/10	**9**	/10
3	/10	**10**	/10
4	/10	**11**	/10
5	/10	**12**	/10
6	/10	**13**	/10
7	/10	**14**	/10
Total			

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

_____
Name (printed)

_____
Signature

_____
Date

1. Let

$$f\left(x\right)=16\sqrt{x}+\frac{32}{\sqrt{x}}-x^2$$

Find the equation of the tangent line at $x=4$.

2. Find the values of a and b so that

$$f(x)=\begin{cases} 2\cos(x-5)+a & \text{if } x<5 \\ b & \text{if } x=5 \\ \dfrac{x^2-3x-10}{x-5} & \text{if } x>5 \end{cases}$$

is continuous at $x=5$?

A) $a=0, b=6$ C) $a=5, b=7$ E) $a=0, b=4$

B) $a=0, b=7$ D) $a=5, b=6$ F) None of the above

3. Compute

$$\lim_{x\to\infty} \sqrt{x^2+3x} - x$$

A) 0

B) $\frac{1}{2}$

C) $\frac{-1}{2}$

D) $\frac{3}{2}$

E) $\frac{-3}{2}$

F) The limit does not exist

4. Let

$f(x) = 3x^2 + 2$

Find $f'(x)$ **using only the definition of the derivative**.

5. Let f and g be two differentiable functions that satisfy

$$f(1)=3 \quad f'(1)=2 \quad f(2)=1 \quad f'(2)=0$$
$$g(1)=5 \quad g'(1)=0 \quad g(2)=2 \quad g'(2)=2$$

Let

$$h(x)=f(x)\arctan(g(x)).$$

Compute $h'(2)$

A) 0

C) $\frac{2}{5}$

E) $\frac{4}{5}$

B) $\frac{1}{5}$

D) $\frac{3}{5}$

F) None of the above

6. The radius of the base of a cone is growing at a rate of 2 in/min. The cone's height is always six times longer than the diameter of the base. How fast is the volume changing when the base's radius is 10 in?

Note: The formula for the volume of a cone with radius r and height h is $V = \frac{\pi}{3} r^2 h.$

A) $12\pi \text{ in}^3/\text{min}$

B) $60\pi \text{ in}^3/\text{min}$

C) $1200\pi \text{ in}^3/\text{min}$

D) $600\pi \text{ in}^3/\text{min}$

E) $2400\pi \text{ in}^3/\text{min}$

F) None of the above

7. Let $f(x) = 16x + \cos(2\pi x)$. If $g(x) = f^{-1}(x)$, compute $g'(17)$.

A) $\frac{1}{16}$

B) $\frac{1}{16-2\pi}$

C) $\frac{-1}{16-2\pi}$

D) $\frac{1}{16+2\pi}$

E) $\frac{-1}{16+2\pi}$

F) None of the above

8. Determine the absolute maximum and absolute minimum for the function

$$f\left(x\right)=3x^4+4x^3-12x^2+1$$

on the interval $-3\le x\le 2$.

9. Let $f(x) = xe^x$

Find the interval(s) where $f(x)$ is concave up and decreasing simultaneously.

10. Evaluate the limit

$$\lim_{x\to 0^+} \frac{\ln\left(\sin\left(2x\right)\right)}{\ln\left(3x\right)}$$

A) 1

B) $\frac{1}{3}$

C) $\frac{2}{3}$

D) 2

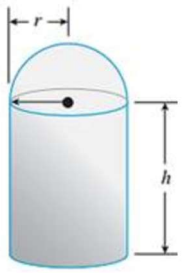
E) The limit does not exist

F) None of the above

11. A silo is to be built which will consist of a cylinder of radius r and height h, and on top will be a hemisphere of the same radius r. The material for the side costs 1/π dollars for square foot while the material for the top hemisphere costs 4/π dollars per square foot. Given that you have 180 dollars in your budget to purchase materials, what dimensions for r and h should you choose to maximize the volume of the silo. In the interest of time, you can skip the work proving that you have an absolute extreme value.

Note: The volume of a hemisphere of radius r is $\frac{2\pi}{3}r^3$ and the surface area is $2\pi r^2$.

The volume of a cylinder of radius r and height h is $\pi r^2 h$ and the surface area of the side is $2\pi rh$ (the surface area of the bases does not get included).



12. Compute

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} 6\sin(3x)+\sqrt{3}\sec^2(2x)\,dx$$

A) $\frac{1}{2}+\frac{\sqrt{3}}{2}$ C) $2+\sqrt{3}$ E) $1+\sqrt{2}$

B) $\frac{\sqrt{2}}{2}+3\sqrt{3}$ D) $\sqrt{2}+\sqrt{3}$ F) None of the above

13. Compute

$$\int_{-1}^{\frac{5}{2}} \frac{8}{\sqrt[3]{2x+3}}\,dx$$

A) 9 C) 27 E) 54

B) 18 D) 36 F) None of the above

14. Find the area in the first quadrant bounded by

$y = \sqrt{x}, y = \frac{-1}{2}x + \frac{3}{2},$ and $y = 0.$

A) $\frac{1}{3}$ C) $\frac{5}{3}$ E) $\frac{7}{3}$

B) $\frac{2}{3}$ D) 2 F) None of the above

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