Math 114
Final Exam Spring 2020

First and Last Name ____________________________(PRINT)   Penn ID__________

This exam has 14 questions, each question is worth 10 points for a total of 140 points. Partial credit will be given for the entire exam so be sure to show all work. On the multiple choice, circle the correct answer and give supporting work, **a correct answer with little or no supporting work will receive little or no credit**. Use the space provided to show all work. A sheet of scrap paper is provided at the end of the exam. If you write on the back of any page please indicate this in some way.

You have **120 minutes** to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Once you have completed the exam, sign the academic integrity statement below.
Do **NOT** write in the grid below. It is for grading purposes only.

Problem	Points	Problem	Points	Problem	Points
1		6		11	
2		7		12	
3		8		13	
4		9		14	
5		10			
Total					

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

Name (printed)	**Signature**	**Date**

1. Find the distance between the centers of the two spheres.

$$x^2 + y^2 + z^2 + 8x = 17 + 2y + 6z$$

$$x^2 + y^2 + z^2 + 11 = 8y + 30z$$

2. Find the point on the plane $x + 2y - 3z - 5 = 0$ that is closest to $(4, 4, -7)$.

Enter the $x-$coordinate of that point.

3. Find the line that intersects the line

$$\begin{array}{ccccc} x & = & 1 & + & t \\ y & = & 2 & - & 3t \\ z & = & & & 2t \end{array}$$

perpendicularly and lies in the plane $x + y - 3z = -1$.

Call the $x-$coordinate of the point where this line intersects the $xy-$plane k. Enter the value of $-8k$.

4. Find the arclength of the curve

$$\mathbf{r}(t) = \left\langle t^2, t - \frac{t^3}{3}, t + \frac{t^3}{3} \right\rangle$$

for $0 \le t \le 3$.

The arclength is $w\sqrt{2}$, Enter the value of w.

5. Find the curvature at time $t=0$ for the curve

$$\mathbf{r}(t)=\left\langle 111+\ln(t+1), \tan t+2t^2+2019, e^t\cos t+\frac{e^\pi}{\sqrt{2}}\right\rangle$$

The curvature is $\dfrac{\sqrt{m}}{3}$. Enter the value for m.

6. Find an equation for the tangent plane to the surface $x^2+xy+y^2+z^2=16$ at the point $(1,2,3)$

This plane intersects the $x-$axis at the point $(p,0,0)$. Enter the value of p.

7. Let $g(x,y,z)=x^2y-2x^3+xyz-6x-7$

Find the directional derivative from the point $(-1,1,1)$ in the direction of the vector $\langle 6,3,-6\rangle$.

Enter the absolute value of this directional derivative.

8. Let

$$g(x,y)=2x^3-6xy-3y^2$$

Find and classify the critical points of $g(x,y)$.

Enter the $y-$coordinate of the local maximum.

9. Find the maximum and minimum values of

$$f(x,y) = 2x + 3y$$

such that $x^2 + xy + y^2 = 84$.

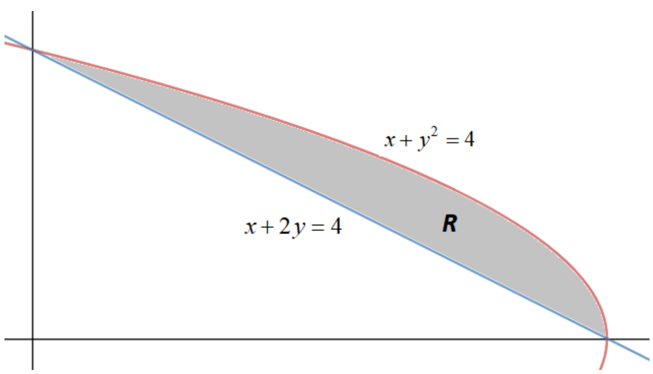
Let $h = |\max - \min|$. **Enter the value of** h.

10. Let

$$\alpha = \iint_R y \, dA$$

Enter the value of 15α when

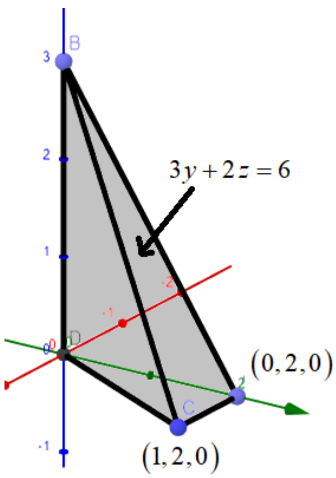
R is the region shown below



11. Let

$$\beta = \iiint_E xy\, dV$$

Enter the value of 100β when E is the solid tetrahedron in the first octant shown below.



12. Evaluate

$$\iiint_D \left(x^2+y^2\right)^2 dV$$

where D is the solid region bounded above by the inverted cone $z = 1-\sqrt{x^2+y^2}$ and below by the $xy-$plane.

The answer is $\frac{\pi}{c}$. **Enter the value of** c.

13. Evaluate

$$\int_C ye^z dx + \left(xe^z + e^y\right) dy + e^z \left(xy + 1\right) dz$$

Where C is the straight line

from $(0,0,0)$ to $\left(\frac{1}{\ln 3}, \ln 3, \ln 2\right)$.

14. Let

$$\omega = \int_C e^x dx + xydy$$

Enter the value of 30ω

when C goes along the

$x-$axis from $(0,0)$ to $(1,0)$,

then along $y = \sqrt{1-x^2}$

to $(0,1)$ and then back to $(0,0)$

along the $y-$axis.