**MATH 114 FINAL EXAM** **Fall 2021** **GREEN FORM**

***This exam consists of problems and pages. You have two hours (120 minutes) to complete the exam.***

*Unless otherwise specified,* ***you must show your work****. It is the work (as opposed to the answers) which will be graded.*

*You should* ***clearly indicate your final answer*** *to each problem.*

*There is ample room to show work. Try not to crowd your work. If you need to use the back of a page,* ***indicate that you are doing so*** *and also* ***indicate which problem the work is for.***

*You may work the problems in any order you wish.*

*If you have a question about* ***what a problem is asking****, raise your hand to ask the proctor.*

**CIRCLE YOUR INSTRUCTOR'S NAME BELOW.**

COOPER GOMEZ QIN ROMERO SHIELDS

**WRITE YOUR NAME HERE and on every page:**

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**WRITE YOUR 8-DIGIT PENN ID NUMBER HERE and on every page:**

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Good luck! You've been preparing all term – you've got this.

1. A piece of wire in space is parametrized by

$$\gamma(t) = \begin{pmatrix} t\sin(t) \\ \frac{2\sqrt{2}}{3}t^{\frac{3}{2}} \\ t\cos(t) \end{pmatrix}, 0 \leq t \leq 1$$

The mass density along the wire is $\rho(x, y, z) = x^2 + y^2 + z^2$.

a. What is the mass of this piece of wire?

b. What is the moment of inertia of this piece of wire about the y-axis?

2. The **kernel** of a matrix $A$ is the collection of all the vectors $\mathbf{v}$ so that $A\mathbf{v} = \mathbf{0}$. (This is a new definition, but don't let that faze you.)

For the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ -1 & -2 & -1 & 3 \\ 0 & 0 & 2 & -6 \end{bmatrix}$$

a. write down the system of linear equations that a vector $\mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ must satisfy so that $\mathbf{v}$ is in the kernel of $A$.

b. Describe the shape of the kernel of $A$: is it a point, a line, a plane. . . ?

3. Consider the plane $P: 3x - 2y - z = 1$ and the surface $S: xy = 5 - z^2$.

The tangent plane to $S$ at $(1,1,2)$ intersects $P$ in a _____________ which can be parametrically described by:

*(fill in the blank and find the parametrization.)*

4. Let $H(x,y) = \begin{bmatrix} x^2 + y \\ e^{x+3} + y^2 \end{bmatrix}$ and $G(u,v) = \begin{bmatrix} -u^2 - 3v \\ vu^2 - e^u \end{bmatrix}$. Write $\begin{bmatrix} r \\ s \end{bmatrix} = H \circ G(u,v)$.
   a. Explain how we know that, nearby $(u,v) = (0,1)$, we can write $u$ and $v$ as functions of $r$ and $s$.

   b. Is the transformation $(u,v) \mapsto (r,s)$ *orientation preserving* or *orientation reversing* nearby to $(0,1)$?

   c. Is the transformation $(u,v) \mapsto (r,s)$ *area increasing or area decreasing* nearby to $(0,1)$?

5. Integrate the 2-form

$$\beta = 3x\, dy \wedge dz + (z^2 - \sin(x))\, dz \wedge dx - 4zx^3\, dx \wedge dy$$

along the surface that bounds the region which is **inside** the cylinder $x^2 + y^2 \leq 9$, **below** the paraboloid $z = 16 - x^2 - y^2$, and **above** the plane $y - z = 1$. This surface is oriented with an **outward** pointing normal.

6. Let $f(x,y,z) = \cos\big(y(e^{x+z} - 1)\big)$.
   a. Compute the degree-5 Taylor polynomial for $f$ at $(0,0,0)$. No need to simplify.

   b. Let $I = (1,2,1)$. What is $D^I f(0,0,0)$?

7. The vector field

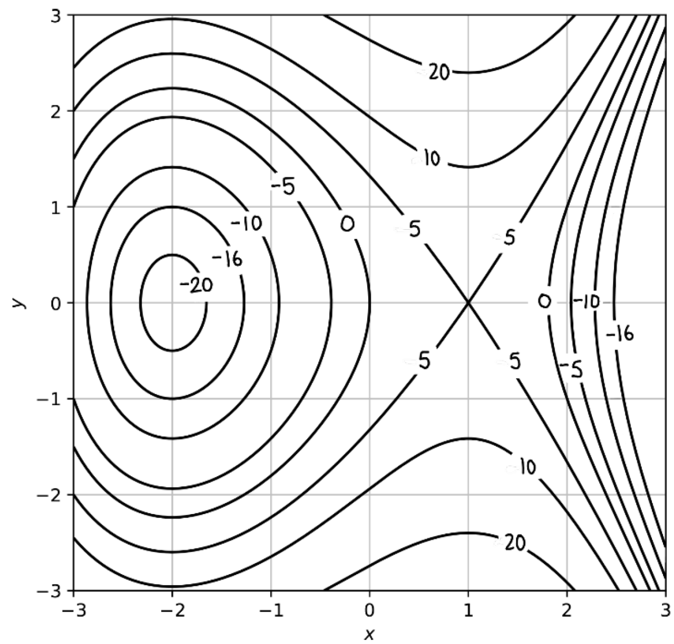
$$F = (x^2 - e^x)\,\hat{\mathbf{i}} - 2xy\,\hat{\mathbf{j}} + z^p e^x\,\hat{\mathbf{k}}$$

   is a curl; that is, $F = \operatorname{curl}(V)$ for some vector field $V$.

   a. What is $p$?

   b. Could it be the case that $V = \nabla f$ for some function $f(x,y,z)$? Explain.

8. This plot shows the level sets for a function $f(x, y)$.




a. This function has two critical points. Find and classify them. Justify your answer.

b. Let $C$ be the straight-line path from $(1,0)$ to $(-2,1)$. What is $\int_C df$?

9. Give $2 \times 2$ matrices which represent the following:

a. Rotate counterclockwise by $\pi$ radians, then stretch vertically by a factor of 3.

b. Horizontal shear which sends the point (0,1) to (4,1), followed by clockwise rotation by $\frac{\pi}{2}$ radians.