MATH 1410 FINAL EXAM DRAFT

VERSION = Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

INSTRUCTIONS:

No book, calculator, or formula sheet. Use a pencil, eraser, and logic.

This is a multiple choice exam.
Some answers may count for partial credit.
Completely wrong answers (random guesses) may be penalized.
Some questions are "select all that apply" – read carefully!

You do not *need* to show your work.
If you do show your work, we may take it into consideration.

Fill in your name, ID, and version on the bubble sheet.
It is important that you fill in the circles completely with a pencil.
If you make a mistake, please erase clearly.

These questions are written carefully: **please do not ask for clarification during the exam.** If anything is unclear, use your best judgement and explain yourself. We will grade accordingly. If we have made a mistake, we'll fix it in the grading.

Cheating, or the appearance of cheating, will be dealt with severely. By placing your name on this page, you agree to abide by the rules.

Should you need it, the formulae for spherical coordinates are:

$$x = \rho\cos\theta\sin\varphi \quad : \quad y = \rho\sin\theta\sin\varphi \quad : \quad z = \rho\cos\varphi$$

Stay calm. All of these problems are doable. You can make it.

1. **PROBLEM:** Compute the work done by the planar vector field

$$\vec{F} = (x^2 - y^3)\hat{\imath} + (2y^2 + x^3)\hat{\jmath}$$

along the circle of radius two centered at the origin, clockwise orientation.

Ⓐ : 0

Ⓑ : 6π

Ⓒ : 12π

Ⓓ : 24π

Ⓔ : None of the above

2. **PROBLEM:** Convert the double integral

$$\int\int (2x+y)x^2y^2\,dx\,dy$$

using coordinates $u = 2x - y$ and $z = x^2y^2$. *Ignore the limits of integration.*

Ⓐ : $\iint uz\,du\,dz$

Ⓑ : $\frac{1}{2}\iint z\,du\,dz$

Ⓒ : $\frac{1}{2}\iint \sqrt{z}\,du\,dz$

Ⓓ : $\iint z\,du\,dz$

Ⓔ : None of the above

3. **PROBLEM:** If the cubic Taylor expansion of $f(u, v)$ about the origin equals

$$f(u,v) = 3 - u^2 + \frac{5}{4}uv - 2v^2 - \frac{1}{3}u^3 + u^2v - \frac{1}{2}uv^2 - \frac{4}{5}v^3 + \cdots$$

what is the value of $\dfrac{\partial^3 f}{\partial u\, \partial^2 v}$ evaluated at the origin?

Ⓐ : 0

Ⓑ : $-1/2$

Ⓒ : -1

Ⓓ : -2

Ⓔ : There is not enough information to say

4. **PROBLEM:** TRUE/FALSE : Select all the statements below which are **TRUE**

Select all that are TRUE.

Ⓐ : The volume element in spherical coordinates is $dV = \rho^2 \sin\phi \; d\theta \, d\phi \, d\rho$

Ⓑ : For two invertible n-by-n matrices A and B: $(AB)^{-1} = A^{-1}B^{-1}$

Ⓒ : The Inverse Function Theorem says that the derivative of the inverse F^{-1} is equal to the inverse of the derivative: $[DF^{-1}] = [DF]^{-1}$

Ⓓ : The flux 1-form of the planar vector field $\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath}$ equals $V_x dy - V_y dx$

Ⓔ : The cross product in 3-D satisfies $\boldsymbol{u} \times \boldsymbol{v} = \boldsymbol{v} \times \boldsymbol{u}$

5. **PROBLEM:** Assume $f: \mathbb{R}^3 \to \mathbb{R}$ and $g: \mathbb{R}^3 \to \mathbb{R}$ are both differentiable. What are the dimensions of the matrix $[Dh]$ representing the derivative of $h = \nabla f \times \nabla g$, where ∇ denotes the gradient and $\times$ denotes the cross product?

Ⓐ : 1 – by – 3
Ⓑ : 3 – by – 1
Ⓒ : 3 – by – 3
Ⓓ : 6 – by – 3
Ⓔ : 3 – by – 6

6. **PROBLEM:** Compute the flux of the vector field

$$\vec{F} = (x - y^2)\hat{\imath} + z^3\hat{\jmath} + (x^2 + y^2 + 2z)\hat{k}$$

across the sphere of radius 2 centered at the point $(0,0,2)$.

Ⓐ : 32π
Ⓑ : 12π
Ⓒ : 4π
Ⓓ : -12π
Ⓔ : None of the above

7. **PROBLEM:** Which of the following is the moment of inertia element dI of the cube rotated about the y-axis, where the cube has each coordinate range from -1 to $+1$, and the density of the cube is given by $x^2 + y^2$.

Ⓐ : $dI = (x^2 + y^2)^2 \, dx \, dy \, dz$
Ⓑ : $dI = (x^2 + z^2)(x^2 + y^2) \, dx \, dy \, dz$
Ⓒ : $dI = (x^2 + z^2)^2 \, dx \, dy \, dz$
Ⓓ : $dI = (x + z)^2(x^2 + y^2) \, dx \, dy \, dz$
Ⓔ : $dI = y^2(x^2 + y^2) \, dx \, dy \, dz$

8. **PROBLEM:** What is the second entry in the vector $(BA)\boldsymbol{x}$, where

$$A = \begin{bmatrix} 1 & 3 & -9 \\ 0 & 2 & 11 \\ 0 & 0 & -4 \end{bmatrix} \quad : \quad B = \begin{bmatrix} 5 & 3 & 14 \\ -2 & -3 & 7 \\ 1 & 4 & -5 \end{bmatrix} \quad : \quad \boldsymbol{x} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$$

Ⓐ : -17

Ⓑ : 8

Ⓒ : 7

Ⓓ : 6

Ⓔ : None of the above

9. **PROBLEM:** Assume that at a point a, the derivative of a function f equals

$$[Df]_a = \begin{bmatrix} -5 & 2 & -1 & 2 \\ 0 & -3 & 1 & 5 \\ -1 & 4 & -3 & 2 \\ 1 & -1 & -1 & -2 \end{bmatrix}$$

If all **inputs** are changing at a rate of $+2$, which output has the largest rate of **decrease**?

Ⓐ : output 1

Ⓑ : output 2

Ⓒ : output 3

Ⓓ : output 4

Ⓔ : There is no unique answer

10. **PROBLEM:** Two functions f and g are linear. If you know that

$$[Df] = \begin{bmatrix} 3 & 8 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad : \quad [D(f \circ g)] = \begin{bmatrix} -2 & -5 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Then, which of the following is the determinant of $[Dg]$?

Ⓐ : $\det[Dg] = 1$
Ⓑ : $\det[Dg] = -1$
Ⓒ : $\det[Dg] = 3$
Ⓓ : $\det[Dg] = 5$
Ⓔ : None of the above; or cannot be determined

11. **PROBLEM:** For which values of constant C is the origin of f a maximum, where

$$f(x,y) = 3x^2y + Cx^2 - 2xy - 4y^2 - 9$$

Ⓐ : $C < 0$

Ⓑ : $C < 4$

Ⓒ : $C < -4$

Ⓓ : $C < -1/4$

Ⓔ : None of the above / depends on x and y

12. **PROBLEM:** Which of the following gives the same result as

$$\int_0^{\pi}\int_0^{2}\int_{4-2x}^{4-x^2} z\cos(xy)\ dz\,dx\,dy$$

Select all that apply.

Ⓐ : $$\int_0^{\pi}\int_{4-2x}^{4-x^2}\int_0^{2} z\cos(xy)\ dx\,dz\,dy$$

Ⓑ : $$\int_0^{2}\int_{4-2x}^{4-x^2}\int_0^{\pi} z\cos(xy)\ dy\,dz\,dx$$

Ⓒ : $$\int_0^{\pi}\int_0^{4}\int_{2-\frac{z}{2}}^{\sqrt{4-z}} z\cos(xy)\ dx\,dz\,dy$$

Ⓓ : $$\int_0^{\pi}\int_0^{2}\int_{2-\frac{z}{2}}^{\sqrt{4-z}} z\cos(xy)\ dx\,dz\,dy$$

Ⓔ : $$\int_0^{\pi}\int_0^{2}\int_{4-2x}^{4-x^2} z\cos(xy)\ dz\,dx\,dy$$

13. **PROBLEM:** When optimizing the function $f = x^2 + 2y^2 + 3z^2$ subject to the constraint $g = 3x - 2y + 5z = 29$, the Lagrange method shows a critical point at $\left(\frac{9}{2}, -\frac{3}{2}, \frac{5}{2}\right)$. What is the value of the Lagrange multiplier at this point?

Ⓐ : 0
Ⓑ : 1
Ⓒ : 2
Ⓓ : 3
Ⓔ : 5

14. **PROBLEM:** What is the probability that a random point in the domain $x^2 + y^2 \leq 4$ chosen with probability density $f = \frac{3}{16\pi}\sqrt{x^2 + y^2}$ has $x^2 + y^2 > 1$?

Ⓐ : $3/4$
Ⓑ : $2/3$
Ⓒ : $5/9$
Ⓓ : $7/8$
Ⓔ : $11/15$

15. **PROBLEM:** For what value of constant C is the function $f = C(x^2 + y^2 + z^2)^{\frac{3}{2}}$ a probability density function on the solid ball of radius 1 centered at the origin?

Ⓐ : $C = 2\pi/3$
Ⓑ : $C = 3/4\pi$
Ⓒ : $C = 4\pi/3$
Ⓓ : $C = 2/\pi$
Ⓔ : None of the above

16. **PROBLEM:** Compute the flux of the curl of the vector field

$$\vec{F} = (x^2 - \sin x)\hat{\imath} + (2y - z^3)\hat{\jmath} + (3z + y^3)\hat{k}$$

across the unit hemisphere $x^2 + y^2 + z^2 = 1$ with $x \geq 0$. *Ignore orientation: all choices below are expressed as absolute values. That is, ignore negative signs in answers.*

Ⓐ : $3\pi/4$
Ⓑ : $3\pi/2$
Ⓒ : 0
Ⓓ : π
Ⓔ : 2π

17. **PROBLEM:** Which pair of vectors has the largest angle between them?

$$\boldsymbol{u} = \begin{pmatrix} 2 \\ -2 \\ 0 \\ -3 \end{pmatrix} \quad : \quad \boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 4 \\ -3 \end{pmatrix} \quad : \quad \boldsymbol{w} = \begin{pmatrix} 0 \\ 1 \\ -2 \\ 3 \end{pmatrix} \quad : \quad \boldsymbol{x} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 3 \end{pmatrix}$$

Ⓐ : $\boldsymbol{u}$ and $\boldsymbol{v}$

Ⓑ : $\boldsymbol{u}$ and $\boldsymbol{w}$

Ⓒ : $\boldsymbol{v}$ and $\boldsymbol{w}$

Ⓓ : $\boldsymbol{v}$ and $\boldsymbol{x}$

Ⓔ : $\boldsymbol{w}$ and $\boldsymbol{x}$

18. **PROBLEM:** Calculate and simplify the product $\alpha \wedge d\alpha$ as much as possible, where

$$\alpha = y^2\, dx - x^2\, dz$$

Ⓐ : $\alpha \wedge d\alpha = 0$

Ⓑ : $\alpha \wedge d\alpha = -x^2\, dx \wedge dy \wedge dz$

Ⓒ : $\alpha \wedge d\alpha = x^2\, dx \wedge dy \wedge dz$

Ⓓ : $\alpha \wedge d\alpha = -2x^2 y\, dx \wedge dy \wedge dz$

Ⓔ : $\alpha \wedge d\alpha = 2x^2 y\, dx \wedge dy \wedge dz$

19. **PROBLEM:** Compute the average, $\bar{r}$, of the function $r = \sqrt{x^2 + y^2}$, on the cone given in cylindrical coordinates by $0 \leq r \leq z$ with $0 \leq z \leq 1$.

Ⓐ : $\bar{r} = 1/2\pi$
Ⓑ : $\bar{r} = 1/2$
Ⓒ : $\bar{r} = 3/2\pi$
Ⓓ : $\bar{r} = 3/5$
Ⓔ : $\bar{r} = \pi/6$

20. **PROBLEM:** What is the determinant of the matrix A, where:

$$A = \begin{bmatrix} 0 & 4 & 3 & -1 & 4 \\ 1 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 2 & 1 & 8 & -3 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

Ⓐ : $\det A = 28$

Ⓑ : $\det A = -28$

Ⓒ : $\det A = 0$

Ⓓ : $\det A = -14$

Ⓔ : $\det A = 56$