Penn ID:

Math 1400 Final Exam 12/19/22

1. Evaluate the indefinite integral $\int 5xe^{2x}\ dx$.

2. Find the local linear approximation to $f(x) = \displaystyle\int_{1/2}^{\frac{3}{2}x^2-1} \frac{\sin \pi\theta}{\theta}\, d\theta$ at $x = 1$.

3. Does $f(x) = x^5 e^{\sinh(x)} \cos(x)$ have a local minimum, local maximum, or neither at the origin?

Penn ID:

Math 1400 Final Exam 12/19/22

4. Consider the following initial value problem $\begin{cases} \dfrac{dx}{dt} = x(1-x) \\ \\ x(0) = x_0 \end{cases}$

   where $x_0 \geq 0$.

   (a) Solve the initial value problem and find $x(t)$ if $x_0 > 0$, and $x_0 \neq 1$.

   (b) If $x_0 = 0$ or $x_0 = 1$ then the initial value problem can be solved without separating. Describe how and find the solutions.

   (c) Compute $\lim_{t\to\infty} x(t)$ (the answer will depend on x_0).

5. Evaluate the indefinite integral $\int \frac{\sqrt{x^2 - 7}}{x}\, dx$.

6. Find the values of p for which each integral converges

   (a) $\displaystyle\int_1^2 \frac{dx}{x(\ln x)^p}$

   (b) $\displaystyle\int_2^\infty \frac{dx}{x(\ln x)^p}$

7. The departure time of an airplane, T, is a continuous random variable taking non-negative values. Suppose the probability that the airplane leaves at a time later than t is $e^{-\lambda t}$ with some positive constant λ.

   (a) Compute the probability density function of T.

   (b) Compute the expected departure time.

8. The curve $x(t) = \cosh(t)$, $y(t) = 2\sinh(t)$, $0 \leq t \leq 1$ is revolved about the line $y = -1$.

   (a) Write down, but **do not evaluate**, an integral expression for the area of the resulting surface.

   (b) Write down, but **do not evaluate**, an expression for the average y-value of the resulting surface.