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1. Suppose that $f(x)$ is an analytic function such that $f(4) = 3$, $f'(4) = -2$, $f''(4) = 5$, and $f'''(4) = 1$. Write the best possible Taylor polynomial centered around $x = 4$ for the function $g(x)$ below, including an appropriate big-O error term.

$$g(x) = (x-4)^2 f(x)$$

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2. Compute the following integral:

$$\int \frac{x^2}{x^2 - 7x + 10} dx.$$

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3. Find the length of the curve given by the graph of

$$y(x) = \int_1^x \sqrt{t^3 - 1}dt$$

between $x = 9$ and $x = 16$.

4. Imagine a very tall cylindrical tank whose radius is 4 meters that is initially empty. Suppose that water is being filled in at the rate of 3 meter$^3$ per minute.

   (a) Calculate how fast the height of the water in the tank must increase.

   (b) At some point during the experiment, it is observed that the actual rate of increase of water's height is only one-third of what we expected. We conclude that there must be a leak. How much water must be leaking per minute?

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5. Evaluate the limit below.

$$\lim_{x \to 0} (1 + 3 \sin x)^{\frac{1}{x}}$$

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6. (a) Write $-2n^3 + 6n^2 - 3n + 1$ as a polynomial in falling powers.

   (b) Find a sequence $b$ such that $\Delta(b)_n = -2n^3 + 6n^2 - 3n + 1$.

   (c) Is it possible to find more than one sequence which answers (b)? Explain why or why not.

7. Find the interval of convergence for the following power series.

$$\sum_{n=1}^{\infty} \cos(\pi n) \frac{n}{n^2+1} (x+2)^n$$

8. Consider the solid obtained by rotating the region between the curves $y = x$, $x^2 + y^2 = 1$, and the $x$-axis in the first quadrant about the $y$-axis.

   (a) Use horizontal cross-sections to set up an integral which evaluates to the volume of the solid.

   (b) A point is randomly and uniformly chosen from the solid from (a). Set up an expression which gives the probability that the $y$-coordinate of the point is greater than $\frac{1}{2}$.

   (c) What is the probability that the $y$ coordinate of our chosen point is $\frac{1}{2}$?

9. The curve $x(t) = \cosh(t)$, $y(t) = 2\sinh(t)$, $0 \leq t \leq 1$ is revolved about the line $y = -1$.

   (a) Write down, but **do not evaluate**, an integral expression for the area of the resulting surface.

   (b) Write down, but **do not evaluate**, an expression for the average $y$-value of the resulting surface.

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