Final Exam - Math 104 - Fall 2018

Each problem is worth 10 points. Circle your answers. Show your work - correct answers with little or no supporting work will receive little or no credit. Partial credit may be given for wrong answers if there is significant progress towards a solution.

Name: (please print) ________________________________

Circle the name of your lecturer: Gressman Haglund Melczer Palvannan Rimmer Sergel Seuffert

TA's Name: (please print) ________________________________

Day of week and time of recitation:________________________

My signature below certifies that I have complied with Penn's Code of Academic Integrity in completing this exam

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Signature

The table below is for grading purposes - do not write below this line

1.__________	7.________
2.__________	8.________
3.__________	9.________
4.__________	10.________
5.__________	11.________
6.__________	12.________

1. Find the constant K so that the function $f(x)$ given by

$$f(x) = \begin{cases} Kxe^{-x/2}, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

is a probability distribution.

(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) 4 (E) $\frac{1}{4}$.

2. Let $y(x)$ be the solution to the initial-value problem

$$x\frac{dy}{dx} = 3y + 2x^3 \ln x\,, \quad x > 0, \quad \text{with } y(e) = e^3\,.$$

Find $y(e^2)$.

(A) $-4e^6$ **(B)** $-e^3$ **(C)** -4

(D) 0 **(E)** 4 **(F)** e^3

(G) $4e^6$

3. Determine which of the following series is convergent.

$$\text{(I)} \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{\frac{1}{n}}, \qquad \text{(II)} \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{\frac{1}{n!}}$$

Justify your reasoning completely.

(A) None of the above. **(B)** Only I **(C)** Only II **(D)** Both I and II.

4. Find the Maclaurin series for $f(x) = \frac{d}{dx}\cos(x^2)$.

(A) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n}}{(2n)!}$

(B) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n-1}}{(2n-1)!}$

(C) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1} \frac{n\,x^{4n-1}}{(2n+1)!}$

(D) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n\,x^{4n-1}}{(2n)!}$

(E) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{2\,x^{4n-1}}{(2n-1)!}$

(F) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1} \frac{x^n}{2\,(2n-1)!}$

(G) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1} \frac{x^{n+1}}{(2n+1)!}$

5. Find the interval of convergence for the power series below.

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n\,3^n}$$

(A) $(-3, 3)$ (B) $[-4, 2)$ (C) $[2, 4]$

(D) $(-\frac{1}{3}, \frac{1}{3})$ (E) $[-\frac{4}{3}, \frac{2}{3})$ (F) $[\frac{2}{3}, \frac{4}{3}]$

(G) $(-\infty, \infty)$

6. The region in the plane bounded above by $y = \sqrt{\sin x}$, below by $y = 0$, and lying between $x = 0$ and $x = \pi$ is revolved around the x-axis. Find the volume of the resulting solid.

(A) 2π

(B) π^2

(C) 4π

(D) 6π

(E) $2\pi^2$

(F) 8π

(G) $8\pi^2$

7. Compute the arc length of the curve below between the endpoints $y = 1$ and $y = 4$.

$$x = \frac{1}{3}y^{-1} + \frac{1}{4}y^3$$

(A) -16 (B) -3 (C) 3

(D) 9 (E) 16 (F) 25

(G) 36

8. Determine whether each series is Convergent (C) or Divergent (D).

I. $\sum_{n=1}^{\infty} \frac{\pi^{n+1}}{3^n \sqrt{n}}$ II. $\sum_{n=2}^{\infty} \frac{\sqrt[3]{n}}{\ln n}$ III. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2}}{2n^2 + 3n - 4}$

(A) I. C, II. C, and III. C **(B)** I. C, II. C, and III. D **(C)** I. C, II. D, and III. D

(D) I. D, II. C, and III. C **(E)** I. D, II. C, and III. D **(F)** I. D, II. D, and III. D

(G) I. C, II. D, and III. C **(H)** I. D, II. D, and III. C

9. Determine whether each series is Absolutely Convergent (AC), Conditionally Convergent (CC), or Divergent (D).

I. $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^n}{ne^n}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{6n^2 + 4n - 5n^3}$

(A) I. AC and II. AC **(B)** I. AC and II. AC **(C)** I. D and II. AC

(D) I. D and II. CC **(E)** I. D and II. D **(F)** I. CC and II. D

(G) I. AC and II. D **(H)** I. CC and II. AC

10. Evaluate

$$\int \cos^5\theta \ \tan^2\theta \, d\theta.$$

(A) $\dfrac{\sin(\theta)^3}{3} - \dfrac{\sin(\theta)^5}{5} + C$

(B) $\dfrac{\sin(\theta)^3}{5} - \dfrac{\sin(\theta)^5}{3} + C$

(C) $\dfrac{\sin(\theta)^2}{2} - \dfrac{\sin(\theta)^4}{4} + C$

(D) $\dfrac{\sin(\theta)^2}{4} - \dfrac{\sin(\theta)^4}{2} + C$

(E) $\dfrac{\cos(\theta)^2}{2} - \dfrac{\cos(\theta)^4}{4} + C$

(F) $\dfrac{\cos(\theta)^3}{3} - \dfrac{\cos(\theta)^5}{5} + C$

11. Evaluate

$$\int_{1/2}^{1} \sqrt{\frac{1-x}{x}}\, dx.$$

Hint: Use the substitution $x = u^2$.

(A) π **(B)** $\pi/4 + 1/2$ **(C)** 2π

(D) $\pi/4 - 1/2$ **(E)** $\pi/2 - 1/4$ **(F)** $1/2$

12. A partial fraction decomposition shows that, up to adding a constant,

$$\int \frac{3x^3 - 2x^2 + 5x + 2}{(x+1)(x-1)^3}\, dx = \frac{P}{(x-1)^2} + \frac{Q}{x-1} + R\ln|x-1| + S\ln|x+1|$$

for real numbers P, Q, R, and S.

Find the real number S. (Hint: To solve the problem, and receive full credit, it is *not necessary* to determine P, Q, or R.)

(A) $S = -2$ (B) $S = -1$ (C) $S = 0$

(D) $S = 1$ (E) $S = 2$ (F) $S = 3$

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