Math 104 Final Exam
Fall 2019

Penn
UNIVERSITY of PENNSYLVANIA

Your Name:

Your TA's Name:

Recitation Number or Day/Time:

Select your Section / Instructor:

MATH 104-001	MWF	10:00–11:00	Frankel
MATH 104-002	TR	10:30–12:00	Villano
MATH 104-003	TR	12:00–1:30	Gressman
MATH 104-004	TR	1:30–3:00	Cooper
MATH 104-005	TR	3:00–4:30	Cooper
MATH 104-007	MWF	1:00–2:00	Rimmer
MATH 104-008	MWF	2:00–3:00	Frankel

Please *turn off and put away all electronic devices*. You may use both sides of an $8.5'' \times 11''$ sheet of paper with handwritten notes. No other resources are allowed. **Show all work.** Please **clearly mark** a multiple choice option for each problem. Remember to fill out the identification information at the top of this page and sign the statement below.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

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Your signature

For Official Use (do not write below this line)

Question Number	Points Possible	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	

1. What is the length of the curve $x = \frac{1}{12}y^3 + \frac{1}{y}$ between $y = 1$ and $y = 2$?

(a) $\frac{5}{6}$ (b) $\frac{7}{6}$ (c) $\frac{3}{2}$ (d) $\frac{11}{6}$ (e) $\frac{13}{6}$ (f) $\frac{13}{12}$

2. A certain random variable X takes values in the range $[0, 1]$ and has probability density function equal to $(2\sqrt{x})^{-1}$ on that interval. Compute the mean μ and find the value of m which satisfies

$$P(0 \leq X \leq m) = \frac{1}{2}.$$

(a) $m = 1/2$, $\mu = 1/3$
(b) $m = 1/2$, $\mu = 1/4$
(c) $m = 1/3$, $\mu = 1/2$
(d) $m = 1/3$, $\mu = 1/4$
(e) $m = 1/4$, $\mu = 1/2$
(f) $m = 1/4$, $\mu = 1/3$

3. Evaluate the integral below.

$$\int \sec^4\theta \, d\theta$$

(a) $\frac{\sec^5\theta}{5} + C$ (b) $\tan^2\theta + C$ (c) $\tan\theta + \frac{\tan^3\theta}{3} + C$ (d) $4\ln(\sec\theta + \tan\theta) + C$ (e) $\frac{1}{3\cos^3\theta} + C$ (f) none of these

4. Determine whether the following series are absolutely convergent (AC), conditionally convergent (CC), or divergent (DV).

$$\text{I.} \quad \sum_{n=1}^{\infty} \frac{(-2)^n (n+1)!}{n^n} \qquad \text{II.} \quad \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

(a) I-AC and II-AC
(b) I-DV and II-CC
(c) I-DV and II-AC
(d) I-AC and II-CC
(e) I-DV and II-DV
(f) I-CC and II-CC

5. Find the interval of convergence of the power series below.

$$\sum_{n=2}^{\infty} \frac{(-1)^n (x-3)^n}{6^n \sqrt[3]{n+1}}$$

(a) $(-3, 9)$ (b) $[-3, 9]$ (c) $(-3, 9]$ (d) $[-3, 9)$ (e) $(-\infty, \infty)$ (f) $\{3\}$

6. Find the volume of the following solid: its base in the xy-plane is the region bounded by $y^2 = x$ and the line $x = 1$; each cross section parallel to the x-axis is a square with one edge in the xy-plane.

(a) $\frac{16}{15}$ (b) $\frac{16\sqrt{2}}{15}$ (c) 4 (d) $4\sqrt{2}$ (e) $\frac{32}{15}$ (f) $\frac{32\sqrt{2}}{15}$

7. Consider the region in the first quadrant bounded by $y = 4/x^3$, $x = 1$, and $y = 1/2$. Find the volume of the solid generated by revolving the region about the line $x = 1/2$.

(a) $\frac{3}{2}\pi$ (b) $\frac{2}{3}\pi$ (c) $\frac{3}{4}\pi$ (d) $\frac{5}{4}\pi$ (e) $\frac{2}{5}\pi$ (f) $\frac{4}{5}\pi$

8. Evaluate the integral below.

$$\int \frac{dx}{x^2\sqrt{9-x^2}}$$

(a) $\dfrac{\sqrt{9-x^2}}{x} + C$

(b) $-\dfrac{\sqrt{9-x^2}}{9x} + C$

(c) $\dfrac{\arcsin(x/3)}{x} + C$

(d) $\dfrac{\csc^2\theta}{3} + C$

(e) $\dfrac{2}{x} + \dfrac{1}{\sqrt{3-x}} - \dfrac{2}{\sqrt{3+x}} + C$

(f) none of these

9. Evaluate the integral below. (Hint: start with a substitution.)

$$\int_0^2 \frac{dx}{1+e^x}$$

(a) $2+\ln\left(\frac{2}{e^2+1}\right)$ (b) $\ln\left(\frac{e^2+1}{2}\right)$ (c) $\ln\left(\frac{e^2+1}{2}\right)$ (d) $\frac{1}{1+e^2}-\frac{1}{2}$ (e) $2-\ln\left(\frac{2}{e^2+1}\right)$ (f) none of these

10. Suppose y solves the initial value problem

$$\frac{dy}{dx} - \frac{e^{-y}}{1+x^2} = 0, \quad y(1) = 2.$$

Compute $\lim_{x\to\infty} y(x)$.

(a) 0 (b) $\ln\left(e^2 - 1 + \pi/2\right)$ (c) $\ln\left(e^2 + \pi/2\right)$ (d) $\ln\left(e^2 + \pi/4\right)$ (e) the limit does not exist (f) none of the above

11. Of the three integrals below, one is proper, one is improper and convergent, and the third is improper and divergent. Determine which is which and justify your answer with an appropriate test for each of the improper integrals.

$$\text{I: } \int_0^1 \frac{e^x}{\sqrt{x}+x^2}\,dx \qquad \text{II: } \int_1^2 \frac{e^x}{\sqrt{x}+x^2}\,dx \qquad \text{III: } \int_2^\infty \frac{e^x}{\sqrt{x}+x^2}\,dx$$

(a) I proper, II converges, III diverges
(b) I proper, II diverges, III converges
(c) I converges, II proper, III diverges
(d) I converges, II diverges, III proper
(e) I diverges, II proper, III converges
(f) I diverges, II converges, III proper

12. Find the quadratic Taylor polynomial for $f(x) = \sqrt{1+x}$ centered at $x = 3$.

(a) $2 - \frac{1}{4}(x-3) - \frac{1}{64}(x-3)^2$

(b) $2 - \frac{1}{4}(x-3) + \frac{1}{32}(x-3)^2$

(c) $2 - \frac{1}{4}(x-3) + \frac{1}{32}(x-3)^2$

(d) $2 + \frac{1}{4}(x-3) - \frac{1}{64}(x-3)^2$

(e) $2 + \frac{1}{4}(x-3) + \frac{1}{64}(x-3)^2$

(f) $2 + \frac{1}{4}(x-3) + \frac{1}{32}(x-3)^2$