Math 104 Finall Examination, version ♢ **May 8, 2018**

Name: _____ **PennID:** _____

Instructor (circle one): Bettiol DeTurck Jang

Instructions

Turn off and put away your cell phone.
Please write your Name and PennID on the top of this page.
Please sign and date the pledge below to comply with the Code of Academic Integrity.
No calculators or any other electronic devices are allowed during this exam.
The only consultation material you may have access to is a 1 page (2 sided) cheat sheet, which you may keep after the exam. You may not consult any other references.
If any question is unclear, raise your hand to ask for clarification.
The regular amount of time you have to complete the exam is 2 hours.

Show your work in the space provided, and then transfer your answers **carefully** *to this sheet by marking your answer with a clearly visible* X *on the table below.*
It is important to show your work because we will be going back over it – you might gain additional partial credit for substantial progress toward the solution of a problem, or you might **lose** *credit for an unsubstantiated correct answer. Good luck!*

#	Answers						#	Answers					
1	(A)	(B)	(C)	(D)	(E)	(F)	9	(A)	(B)	(C)	(D)	(E)	(F)
2	(A)	(B)	(C)	(D)	(E)	(F)	10	(A)	(B)	(C)	(D)	(E)	(F)
3	(A)	(B)	(C)	(D)	(E)	(F)	11	(A)	(B)	(C)	(D)	(E)	(F)
4	(A)	(B)	(C)	(D)	(E)	(F)	12	(A)	(B)	(C)	(D)	(E)	(F)
5	(A)	(B)	(C)	(D)	(E)	(F)	13	(A)	(B)	(C)	(D)	(E)	(F)
6	(A)	(B)	(C)	(D)	(E)	(F)	14	(A)	(B)	(C)	(D)	(E)	(F)
7	(A)	(B)	(C)	(D)	(E)	(F)	15	(A)	(B)	(C)	(D)	(E)	(F)
8	(A)	(B)	(C)	(D)	(E)	(F)							

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

_____ _____

Signature Date

1. The base of a solid S is the region bounded by the graphs of $y = x^2$ and $y = x$. The cross-sections perpendicular to the x-axis are squares. Compute the volume of the solid S.

(A) 1

(B) $\frac{2}{7}$

(C) $\frac{4}{15}$

(D) $\frac{2}{21}$

(E) $\frac{1}{30}$

(F) 2

2. The region between the graph of $y = 1 - x^2$ and the x-axis is rotated around the line $y = 2$. What is the volume of the resulting solid?

(A) 3π

(B) 4π

(C) $\frac{8\pi}{3}$

(D) $\frac{64\pi}{15}$

(E) $\frac{16\pi}{5}$

(F) 32π

3. Find the length of the curve

$$x = \frac{y^3}{6} + \frac{1}{2y}, \qquad 1 \leqslant y \leqslant 2$$

(A) $\frac{17}{12}$

(B) 1

(C) $\frac{8}{7}$

(D) $\frac{13}{9}$

(E) 2

(F) $\frac{11}{8}$

4. Solve the initial-value problem

$$x\frac{dy}{dx} + y = \pi \sin(\pi x), \qquad y(1) = 1$$

and find $y(2)$.

(A) $y(2) = 0$

(B) $y(2) = 1$

(C) $y(2) = -\pi$

(D) $y(2) = \frac{2}{3}$

(E) $y(2) = 2\pi$

(F) $y(2) = -\frac{1}{2}$

5. A tank initially contains 100 gallons of brine in which 50 lb of salt are dissolved. A brine containing 1 lb/gal of salt runs into the tank at the rate of 10 gallons per minute. During the process, the tank is kept well-mixed and the resulting salt water flows out of the tank at the rate of 10 gallons per minute. How long will it take the amount of salt in the tank to reach 80 lb?

(A) 50 minutes

(B) 80 minutes

(C) 100 minutes

(D) $10 \ln \frac{5}{2}$ minutes

(E) $20 \ln \frac{4}{3}$ minutes

(F) $30 \ln 2$ minutes

6. $\displaystyle\int_0^{\pi/2} \cos^5 x \, dx =$

(A) $\frac{1}{5}$

(B) $\frac{8}{15}$

(C) $\frac{2}{3}$

(D) $\frac{6}{5}$

(E) $\frac{5}{3}$

(F) $\frac{28}{15}$

7. $\int_0^\infty (x+1)e^{-x}\,dx =$

(A) $e-1$

(B) $2e-2$

(C) $2e$

(D) 1

(E) 2

(F) The integral diverges.

8. $\displaystyle\int_0^1 \frac{x+1}{(x+2)(x+3)}\,dx =$

(A) $4\ln 2 - 2\ln 3$

(B) $4\ln 3 - 2\ln 2$

(C) $5\ln 2 - 3\ln 3$

(D) $5\ln 3 - 3\ln 2$

(E) $3\ln 3 - \ln 2$

(F) $3\ln 2 - \ln 3$

9. $\int_1^e 16x^3 \ln x \, dx =$

(A) $4e^4$

(B) $4e^4 - 1$

(C) $4e^4 + 1$

(D) $3e^4$

(E) $3e^4 - 1$

(F) $3e^4 + 1$

10. For any $n > 0$, the function $f(x) = \begin{cases} (n+1)x^n & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ is a probability distribution (on the interval $[0, 1]$). For which value of n is the mean of this distribution equal to $3/4$?

(A) $1/3$

(B) $1/2$

(C) $2/3$

(D) $3/4$

(E) 1

(F) 2

11. Find the sum $A + B + C$, where

$$A = \lim_{n \to +\infty} \frac{n \cos n}{2n^2 + 1}, \qquad B = \lim_{n \to +\infty} \left(1 + \frac{2}{n}\right)^{n+1}, \qquad C = \lim_{n \to +\infty} \frac{3^n + 2n}{5^n}$$

(A) e^2

(B) $\frac{1}{2} + e^2$

(C) $\frac{3}{5} + e$

(D) $\frac{2}{5} + e$

(E) 2

(F) e

12. Consider the following infinite series:

$$A = \sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2}, \qquad B = \sum_{n=1}^{\infty} \frac{3n}{\pi n + 1}, \qquad C = \sum_{n=1}^{\infty} \left(\sin\left(\frac{2\pi\sqrt{n}}{n+1} \right) \right)^n$$

(A) A diverges, B and C converge

(B) B diverges, A and C converge

(C) C diverges, A and B converge

(D) A, B, and C diverge

(E) A, B, and C converge

(F) None of the above

13. A basketball falls vertically off the edge of a table, and rebounds 80% of its previous height after each bounce. If the ball travels a total distance of 9 m until it stops moving, how high above the floor is the table top from which it fell?

(A) 1 m

(B) 2 m

(C) 3 m

(D) 4 m

(E) 5 m

(F) None of the above

14. Find the (largest) interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{\pi^2 \, x^n}{2^{n+1}}$$

(A) $[-2, 2]$

(B) $[-2, 2)$

(C) $(-2, 2)$

(D) $(-2, 2]$

(E) Only $x = 0$

(F) All real numbers

15. Use the Taylor Polynomial of degree 4 of $f(x) = \ln(x+1)$ centered at $x_0 = 0$ to approximate the value of $\ln(2)$.

(A) $\ln(2) \approx 7/12$

(B) $\ln(2) \approx 5/6$

(C) $\ln(2) \approx -4/3$

(D) $\ln(2) \approx 4/3$

(E) $\ln(2) \approx 5/12$

(F) $\ln(2) \approx -1/24$