Name: _____ Name of instructor: _____

Final Exam for Math 104, Spring 2019

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
Total	140	

- Please show **ALL** your work on this exam paper. No credit will be given without work.
- **NO** books, class notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam.
- You **ARE** allowed the formula sheet we provided with your own notes added in the white spaces.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this final examination.

Name (printed): _____

Signature: _____ Date: _____

1. Find the volume of the solid generated by rotating the region bounded by $x = 2$, $x = 3$, $y = 0$ and $y = \frac{x}{4-x}$ about the line $x = 4$.

(A) π **(B)** 2π **(C)** 3π

(D) 4π **(E)** 5π **(F)** 6π

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2. Find the arc length of the curve defined by

$$f(x) = \int_1^x \sqrt{\frac{144}{t^2(1+t)^2} - 1} \; dt$$

for $1 \leq x \leq 2$.

(A) $12\ln(2) - 24\ln(3)$

(B) $24\ln(2) - 12\ln(3)$

(C) $6\ln(2) - \ln(3)$

(D) $\ln(2) - 6\ln(3)$

(E) $\ln(2) + \ln(3)$

(F) $\ln(2) - \ln(3)$

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3. A thin plate is defined by the region between the x-axis and the curve

$$y = \frac{1}{(1+x^2)^{3/2}}$$

for $0 \leq x \leq 1$. Find the x-coordinate of the centroid of this thin plate.

(A) $\sqrt{2}+1$ **(B)** $\sqrt{2}+2$ **(C)** $\sqrt{3}+1$

(D) 3 **(E)** $\sqrt{2}-2$ **(F)** $\sqrt{2}-1$

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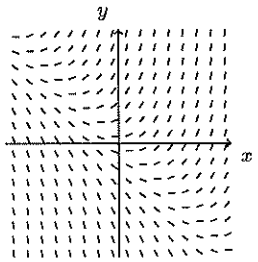
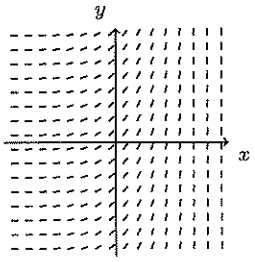
4. Match the differential equations (α, β, γ) and slope fields (i,ii,iii) below. To receive full credit you must justify your answers.

$$(\alpha)\ \frac{dy}{dx} = x^2 - y \qquad (\beta)\ \frac{dy}{dx} = x + y \qquad (\gamma)\ \frac{dy}{dx} = e^x$$

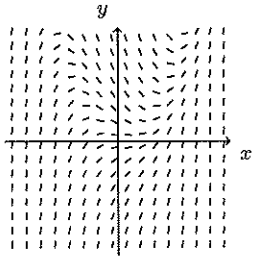
(i)



(ii)



(iii)



(A) i=α, ii=β, iii=γ (B) i=α, ii=γ, iii=β

(C) i=β, ii=α, iii=γ (D) i=β, ii=γ, iii=α

(E) i=γ, ii=α, iii=β (F) i=γ, ii=β, iii=α

5. Let $y(x)$ be the solution to the initial value problem

$$\sqrt{1-x^2}\,\frac{dy}{dx} = xy, \quad y(0) = 1$$

Compute $y(\frac{1}{2})$.

(A) $e^{\left(1-\sqrt{\frac{1}{2}}\right)}$ (B) $e^{\left(1-\frac{\sqrt{3}}{2}\right)}$ (C) $e^{\left(\frac{\sqrt{3}}{2}\right)}$ (D) $e^{\left(\frac{\sqrt{1}}{2}\right)}$ (E) $e^{\left(1-\frac{3}{4}\right)}$ (F) $e^{\left(1-\frac{1}{2}\right)}$

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6. Let $y(x)$ be the solution to the initial value problem

$$x\frac{dy}{dx} + 2y = \sin(x), \quad y(\pi) = 0.$$

Compute $y(2\pi)$.

(A) -3π (B) $-\frac{3}{4\pi}$ (C) $-\frac{2}{3\pi^2}$ (D) 2π (E) $\frac{1}{4\pi}$ (F) $\frac{3}{2\pi^2}$

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7. Evaluate the **improper** integral

$$\int_0^1 x \ln(x^2)\, dx$$

(A) 2 **(B)** -3 **(C)** 4

(D) $-\frac{1}{2}$ **(E)** $\frac{1}{3}$ **(F)** $-\frac{1}{4}$

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8. Evaluate the integral

$$\int \frac{x^2+7x+9}{(x+1)^2(x+2)}\,dx$$

(A) $\ln|x+1| - 2\ln|x+2| + 3(x+1)^{-1} + C$

(B) $3\ln|x+1| + 4\ln|x+2| + (x+1)^{-1} + C$

(C) $4\ln|x+1| - 3\ln|x+2| + C$

(D) $2\ln|x+1| - \ln|x+2| - 3(x+1)^{-1} + C$

(E) $2\ln|x+1| - 3(x+2)^{-2} + C$

(F) $-\ln|x+1| - \ln|x+2| + 2(x+2) + C$

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9. Find the constant k so that the function

$$f(x) = k(\sin^3 x \cos^2 x)$$

is a probability density function on $[0, \pi]$.

(A) $\frac{2}{5}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{4}{9}$

(D) $\frac{11}{2}$ **(E)** $\frac{13}{3}$ **(F)** $\frac{15}{4}$

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10. Determine whether the sequence below converges or diverges and find its limit.

$$a_n = n\left(\ln(n+2) - \ln(n)\right)$$

(A) converges, $\lim_{n\to\infty} a_n = 0$

(B) diverges, $\lim_{n\to\infty} a_n = +\infty$

(C) converges, $\lim_{n\to\infty} a_n = e^{-2}$

(D) diverges, $\lim_{n\to\infty} a_n = -\infty$

(E) converges, $\lim_{n\to\infty} a_n = 2$

(F) diverges, $\lim_{n\to\infty} a_n$ does not exist

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11. In the following series, determine whether the series diverges, converges absolutely, or converges conditionally (circle one for each). **Show your reasoning for each series.**

$$A)\ \sum_{n=1}^{\infty} \frac{(\ln n)^{10}}{n^2} \qquad B)\ \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n+\arctan(n)} \qquad C)\ \sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$$

A) Diverges, Converges Absolutely, Converges Conditionally

B) Diverges, Converges Absolutely, Converges Conditionally

C) Diverges, Converges Absolutely, Converges Conditionally

12. Calculate

$$\lim_{x \to 0} \frac{\cos(x^{24}) \sin(x^{13}) - x^{13}}{x^{39}}$$

(A) $1/13$ **(B)** $1/2$ **(C)** $1/6$

(D) $-1/2$ **(E)** $-1/6$ **(F)** 0

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13. Using a series, estimate the value of

$$\int_0^1 \sin(x^2)dx$$

to within .001.

(A) 1/3 **(B)** 5/6 **(C)** 13/42

(D) 101/120 **(E)** -1 **(F)** 1/2

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14. For values of x does the power series

$$\sum_{n=1}^{\infty} \frac{(3x-5)^n}{n^2}$$

converge?

(A) $[4/3, 2)$ **(B)** $[4/3, 2]$ **(C)** $[-4/3, 14/3)$

(D) $[-4/3, 14/3]$ **(E)** $[-1, 1)$ **(F)** $[-1, 1]$
